R&S®FSWP VALIDITY OF POSITIVE PHASE NOISE VALUES

White paper | Version 01.00

ROHDE&SCHWARZ

Make ideas real

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1 Background

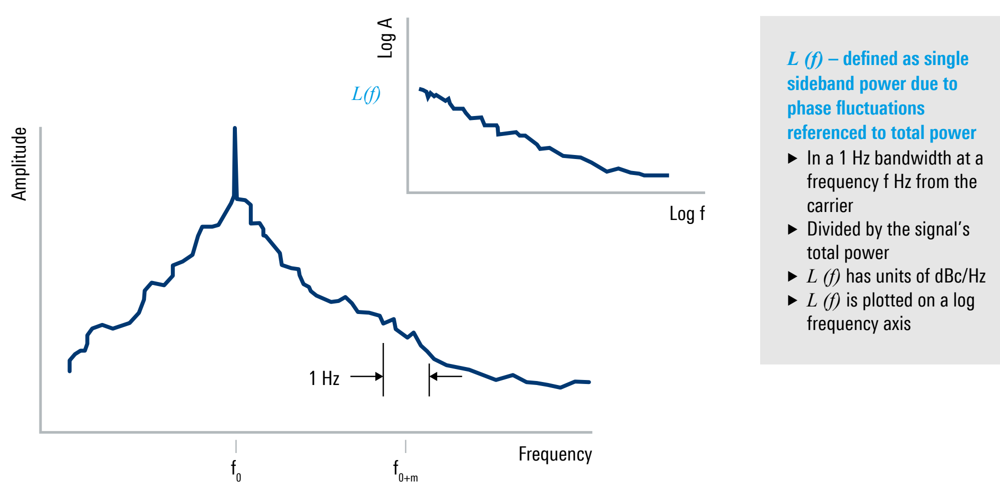
With recent revisions to the R&S®FSWP data sheet (as of Version 07.00), the phase noise sensitivity table for the B61 option has been extended to include specifications for offset frequencies of 0.01 and 0.1 Hz. This change has prompted questions from several R&S®FSWP users regarding positive (greater than zero dBc/Hz) values in the 0.01 Hz column for carrier frequencies above 1 GHz. See excerpt below:

Phase noise sensitivity with R&S®FSWP-B61 cross correlation (low phase noise) option

Specified values in dBc (1 Hz). For typical values subtract 6 dB.											
RF input frequency	**Offset frequency from the carrier**										
	0.01 Hz	0.1 Hz	1 Hz	10 Hz	100 Hz	1 kHz	10 kHz	100 kHz	1 MHz	10 MHz	30 MHz
1 MHz	–60	–105	–118	–136	–148	–166	–176	–176			
10 MHz	–40	–86	–115	–132	–142	–160	–170	–170	–170		
100 Mhz	–20	–66	–95	–117	–140	–166	–170	–173	–175	–175	–175
1 GHz	0	–46	–75	–97	–120	–150	–166	–173	–173	–173	–173
3 GHz	**+10**	–36	–65	–87	–110	–140	–156	–158	–163	–170	–170
7 GHz	**+17**	–29	–58	–80	–103	–133	–152	–153	–157	–166	–166
10 GHz	**+20**	–26	–55	–77	–100	–133	–152	–153	–157	–173	–175
16 GHz	**+24**	–22	–51	–73	–96	–129	–148	–149	–153	–170	–171
26 GHz	**+28**	–18	–47	–69	–92	–125	–144	–145	–149	–166	–167
50 GHz	**+34**	–12	–41	–63	–86	–119	–138	–139	–143	–160	–161

For offset frequencies ≥ 1 Hz, start offset = 1 Hz; for offset frequencies < 1 Hz, start offset = 0.01 Hz.
Correlation factor = 1, internal frequency reference, 30 Hz internal reference loop bandwidth, signal level ≥ 10 dBm. For instruments with R&S®FSWP-B64 option: signal source output = off. For sensitivity with signal source = on, see section on R&S®FSWP-B64 additive phase noise measurements.

The basis of these questions stems from an obsolete spectrum analyzer based definition of single sideband phase noise *L(f)*, where *L(f)* was defined as the spectral power density of the noise sidebands in a 1 Hz band at an offset frequency (f) away from the carrier divided by the total signal power.



In equation form:

$$L(f) = \frac{\text{Area of 1 Hz bandwidth}}{\text{Total area under the curve}}$$

This old definition of *L(f)* results in confusion when measuring carrier noise with large angle phase fluctuations and has been corrected by the IEEE using more modern measurement methods [1].

[1] IEEE Standards Coordinating Committee 27 on Time and Frequency. IEEE Standard 1139-2008, Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology – Random Instabilities. New York, New York: IEEE, 2009

This paper reviews the classical spectrum analyzer measurement of *L(f)* and its shortcomings and discusses how modern phase noise test sets measure phase noise and avoid the limitations imposed by direct spectrum measurements.

The legacy definition of single sideband phase noise is particularly well suited for phase noise measurements made using a traditional swept spectrum analyzer. This measurement technique is known as a direct phase noise measurement. In this case, the above equation is generally simplified by assuming the total signal power is equal to the carrier power.

In dB form, this relationship is:

$$L(f) = P_n \text{ (1 Hz noise power density in dBm/Hz)} - P_s \text{ (carrier power in dBm)}$$

The following figure illustrates this classical spectrum analyzer measurement:

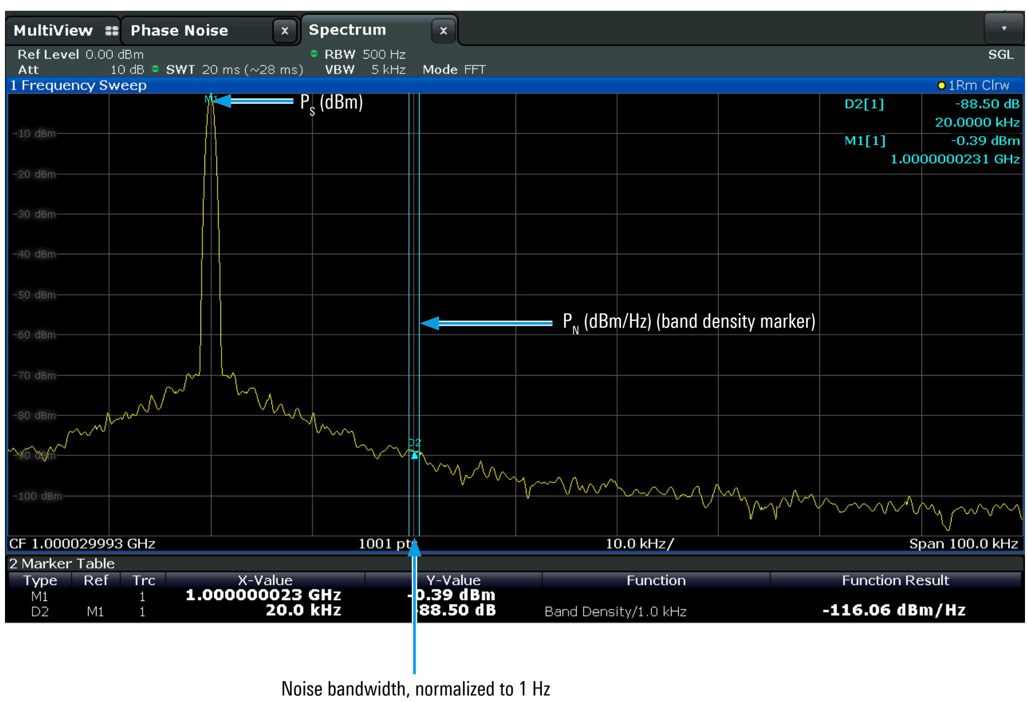


Fig. 1. Direct phase noise measurement using a spectrum analyzer

In most situations, this assumption works quite well since for the majority of signals with low phase noise, the power in the noise sidebands is much smaller than that of the carrier. However, for signals with high levels of phase noise, this measurement technique is no longer valid because phase modulation with large phase variations spreads the energy into higher order sidebands and reduces the power of the carrier.

Since phase noise is phase modulation (PM) of a CW carrier with noise as the modulating signal, we can relate this to the simple case of classical CW phase modulation. As a review of phase modulation and its derivative frequency modulation, the PM frequency spectrum has an infinite number of sidebands, unlike AM.

The amplitude of each of these sidebands can be determined using the Bessel function of the first kind $J_n(m_\varphi)$, where m_φ is the PM phase deviation in radian (rad).

The following figure represents the relationship between the carrier and each of its sidebands for phase modulation.

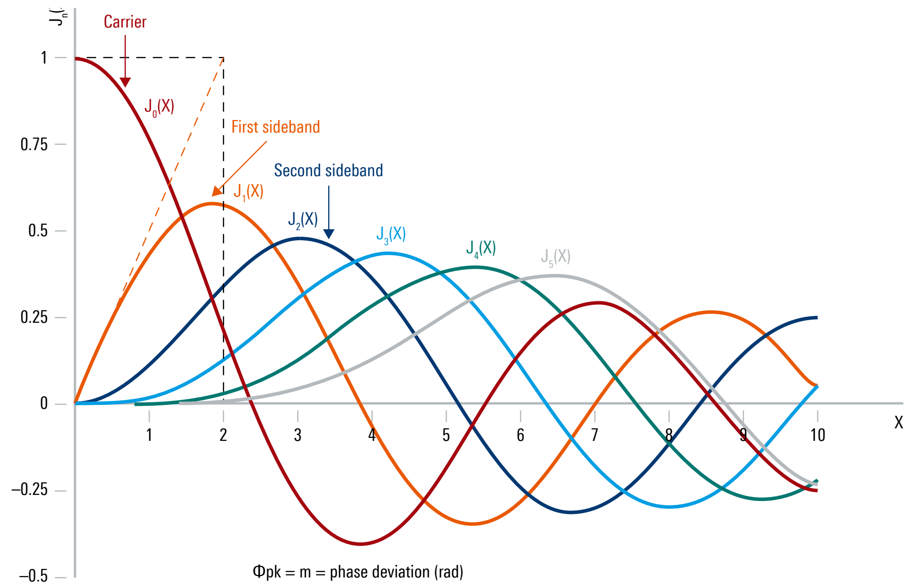


Fig. 2. Bessel functions showing the relationship of sideband voltage to φ_{pk} for phase modulation

The horizontal axis of the above graph represents the peak phase deviation of the carrier (φ_{pk}) in rad, sometimes designated as m or β, and the vertical axis shows the amplitude of the carrier ($J_{0(\varphi pk)}$) and its sidebands (J_1 through J_n) in volt (V).

As shown above, for small values of φ_{pk}, the amplitude of the first sideband is nearly linear with a slope of 0.5 and a carrier amplitude of 1. The ratio of the first sideband voltage to carrier voltage is then equal to half the peak phase deviation.

$$\frac{V_{SB}}{V_C} = \frac{1}{2}\varphi_{pk}\left(rad\right)$$

As φ_{pk} increases, the levels of the first and second sidebands increase and eventually, as φ_{pk} reaches approximately 2.4 rad, the carrier voltage is completely extinguished.

Squaring the above equation allows us to relate the phase modulation first sideband power to carrier power as follows:

$$\frac{P_{SB}}{P_C} = \left[\frac{V_{SB}}{V_C}\right]^2 = \left[\frac{1}{2}\varphi_{pk}\left(rad\right)\right]^2 = \frac{1}{4}\varphi^2{}_{pk}\left(rad\right)^2$$

With $\varphi_{pk} \leq 0.2$ rad, phase modulation is considered to be small angle or narrowband modulation and the ratio of the first sideband power to carrier power conforms to the equation above. These concepts can be demonstrated with a signal generator and spectrum analyzer, as shown in the following figures.

For the figure below, φ_{pk} = 0.01 rad and the calculated ratio of first sideband power to carrier power expressed in dB form is –46.02 dB, which closely corresponds to the measured value of –46.07 dB.

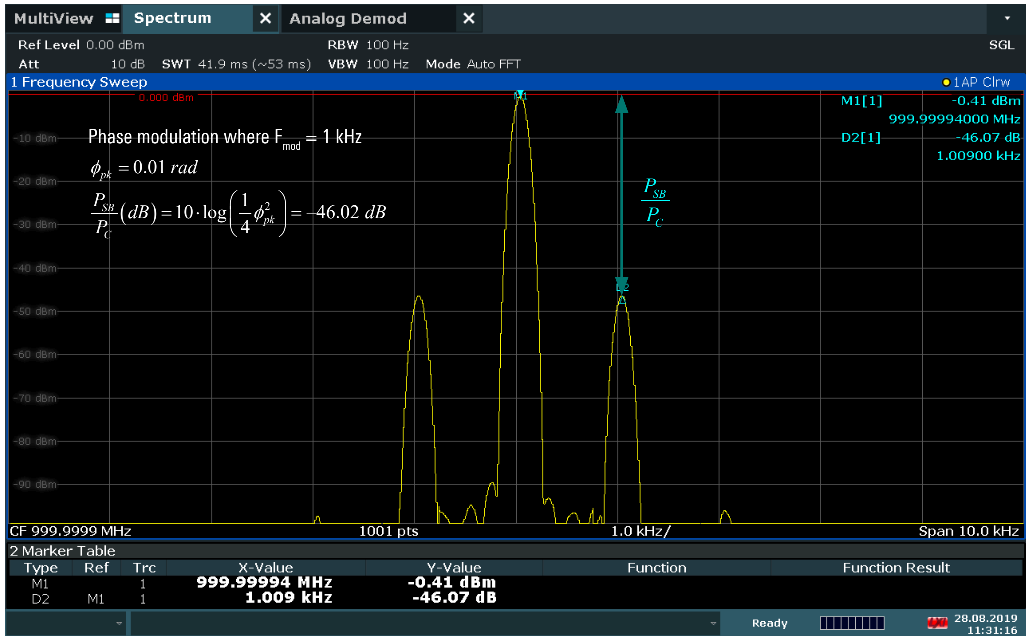


Fig. 3. Spectrum analyzer display of phase modulation where φ_{pk} = 0.01 rad

The following figure shows the changes in the RF spectrum when the phase deviation is increased to 0.1 rad.

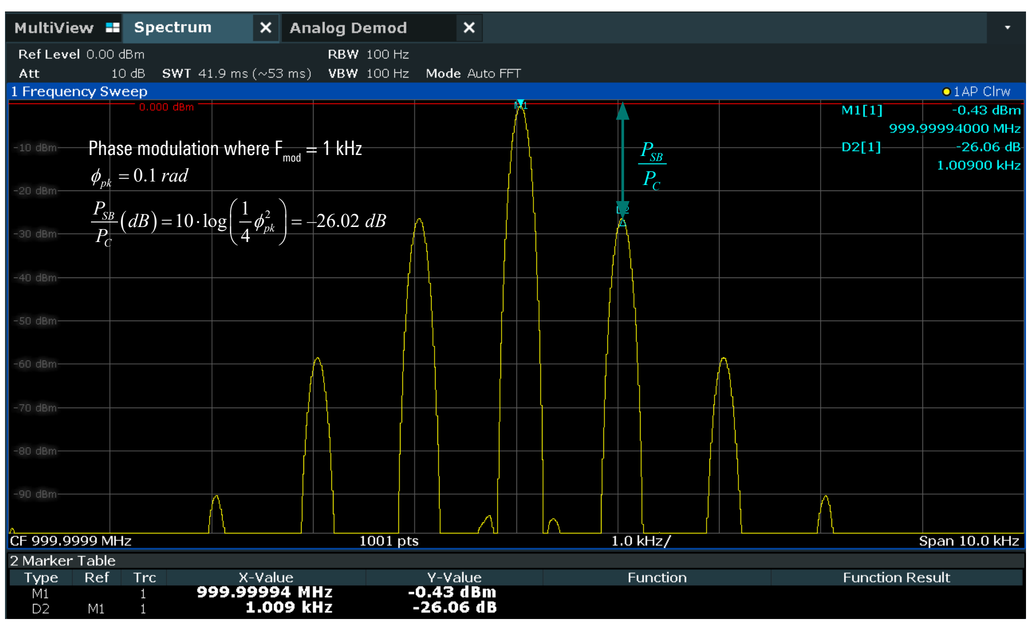


Fig. 4. Spectrum analyzer display of phase modulation where φ_{pk} = 0.1 rad

Notice in the above spectrum analyzer display that when the phase deviation is increased to 0.1 rad, the calculated ratio of first sideband power to carrier power decreases to –26.02 dB and still closely corresponds to the measured value of –26.06 dB. Also, additional modulation sidebands become visible, as predicted by the Bessel function graph shown in Fig. 2.

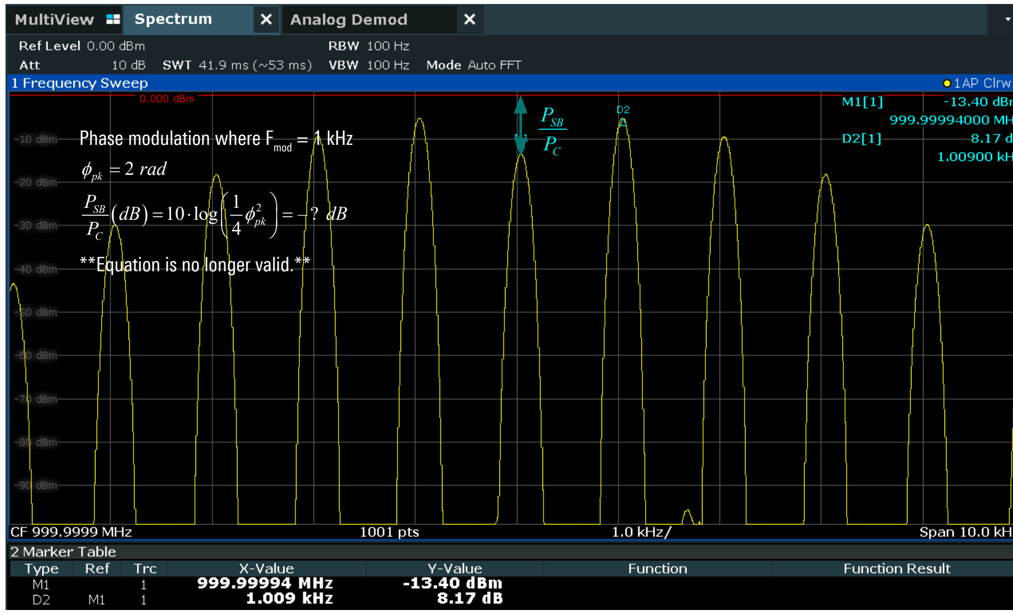


Fig. 5. Spectrum analyzer display of phase modulation where $\varphi_{pk} = 2$ rad

In the above figure, the phase modulation deviation was increased to 2 rad and our equation for first sideband power to carrier power is no longer valid.

Additionally, even more modulation sidebands appear in the spectrum and the carrier power starts to decrease, again as predicted by the Bessel function graph shown in Fig. 2.

The above explanation and spectrum analyzer examples demonstrate that the classical definition of phase noise based on the ratio of single sideband spectral density to carrier power only applies for small angle phase noise and therefore a more general definition is required. In 2009, the IEEE developed a general definition of phase noise that satisfies both small angle and large angle phase noise (see footnote 1).

Phase noise measurement techniques that conform with the IEEE definition are described in section 2 below.

2 Phase detector based phase noise test sets

Most phase noise test sets utilize a double balanced mixer as a phase detector. The output of the oscillator under test is applied to one of the mixer's inputs and a reference oscillator with much lower phase noise is applied to the mixer's remaining input.

The reference oscillator is maintained in quadrature by using a phase locked loop, forcing the DC component of the mixer's output to 0 V. The mixer's output is then lowpass filtered, resulting in a voltage that is proportional to the carrier's phase fluctuations. This signal is then digitized by an analog to digital converter (ADC).

The digitized time samples are converted to the frequency domain using fast Fourier transform (FFT) and then the power spectral density is calculated, resulting in the spectral density of phase fluctuations $S_{\varphi}(f)$

where

$$S_{\varphi}(f) = \frac{\Delta\varphi^2 RMS(f)}{B}\left[\frac{rad^2}{Hz}\right]$$

where $\Delta\varphi$ represents the phase excursion around the carrier and B is the noise bandwidth.

Historical spectrum analyzer measurements of $L(f)$ using the direct phase noise method measure phase noise when the phase variation is much less than 1 rad; however, phase noise measurement systems measure $S_{\varphi}(f)$ and mathematically convert to $L(f)$, which allows the phase variation to exceed the small angle restriction.

The graph below shows that the typical limit for the small angle criterion is a line with a slope of –10 dB/decade that passes through a 1 Hz offset at –30 dBc/Hz. This represents a peak phase deviation of approximately 0.2 rad integrated over any one decade of offset frequency.

$$L(f) = \frac{S_{\phi}(f)}{2}$$

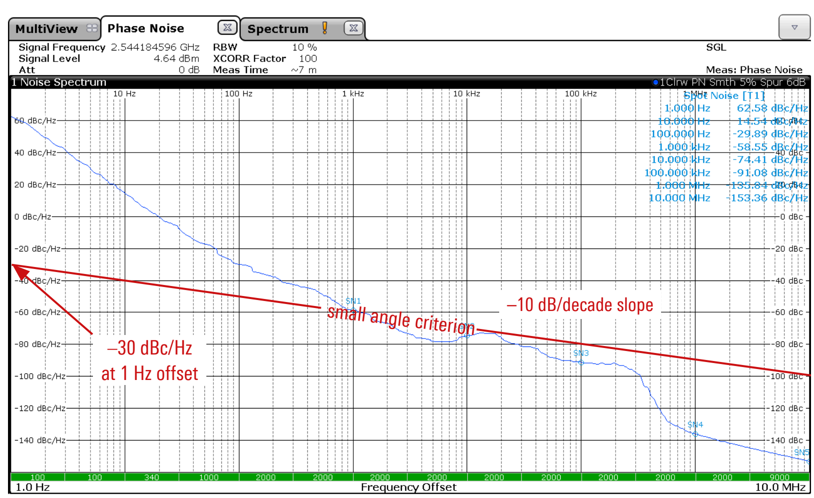


Figure 6. Phase noise measurement showing the confusion resulting from the old definition of phase noise

This plot of $L(f)$, resulting from the phase noise of a free-running VCO, illustrates the confusing display of measured results that can occur if the instantaneous phase modulation exceeds a small angle.

Measured data $S_\Phi(f)/2$ is correct, but historical $L(f)$ is obviously not an appropriate data representation since it reaches +15 dBc/Hz at a 10 Hz offset (15 dB more power at a 10 Hz offset than the total power in the signal). The new IEEE definition of $L(f) = S_\Phi(f)/2$ allows this condition since $S_\Phi(f)$ in dB form is relative to 1 rad. Exceeding 0 dB simply means that the measured phase variations are greater than 1 rad. The IEEE standard states:

"This redefinition is intended to avoid difficulties in the use of $L(f)$ in situations where the small angle approximation is not valid. $L(f)$, as defined, should be designated as the standard measure of phase instability in the frequency domain.

The reasons are the following:

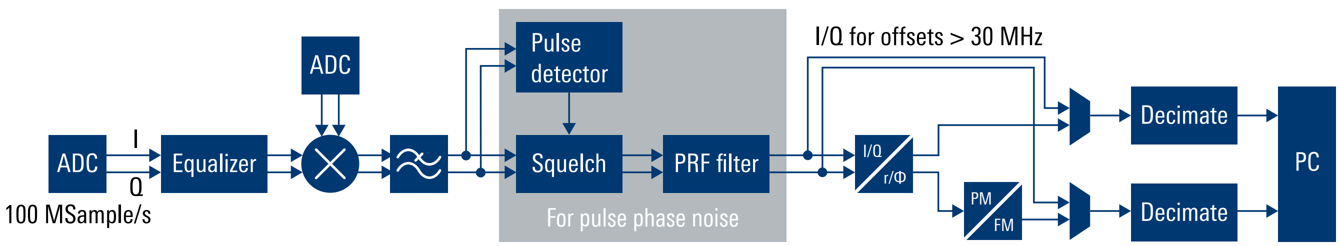
- It can always be measured unambiguously, and
- It conforms to the prevailing usage."

Relating $L(f)$ and $S_{\Phi(f)}$ in dB form, $L(f) = S_{\Phi(f)(dB)} - 3$ dB.

3 R&S®FSWP signal processing

The R&S®FSWP uses a modern DSP based digital frequency discriminator to measure phase noise. The RF signal under test is mixed down to the baseband or a very low IF frequency using I/Q mixers. The mixer outputs are then digitized by four 100 Msample/s ADCs, using a dedicated ADC for each of the mixer's I and Q outputs (channels 1 and 2).

After the ADCs, the I and Q bit streams are processed into phase (PM noise) and magnitude (AM noise) signals using digital signal processing. The phase portion (PM) of the signal is then converted to frequency (FM or Δf(t)) to remove phase wrapping caused by slight differences between the DUT frequency and that of the internal local oscillators in the R&S®FSWP.



Following decimation and fast Fourier transform of the frequency information, a power spectral density is calculated to obtain the spectral density of frequency fluctuations $S_v(f)$. $S_v(f)$ can easily be converted into the spectral density of phase fluctuations $S_\varphi(f)$ by dividing by f^2, where f is the offset frequency from the carrier.

Single sideband phase noise *L(f)* can now be calculated using the IEEE definition by simply dividing by 2. For a more detailed explanation of R&S®FSWP signal processing, see [1].

[1] Rohde & Schwarz, 2-Port Residual Phase Noise Measurements. 2017; Application Note, No. 1EF100_2e, Munich Germany

4 Conclusions

The R&S®FSWP uses a modern DSP based digital frequency discriminator to measure the spectral density of frequency fluctuations, which are then mathematically converted to single sideband phase noise $L(f)$ based on the IEEE definition of phase noise. Just as in the case of phase detector based systems, the digital frequency discriminator used in R&S®FSWP is not limited to small angle phase fluctuations and provides valid measurements of phase noise even for large angle phase fluctuations.

The specified values of phase noise sensitivity published in the R&S®FSWP data sheet, as referenced on page 1, are therefore valid – even those that are greater than 0 dBc/Hz.

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White paper | R&S®FSWP Validity of Positive Phase Noise Values
Data without tolerance limits is not binding | Subject to change

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