R&S ®GSACSM
Signal Parameters
Calculation Basis

1179686602
Version 01

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1 Introduction

To be able to transmit digital data, such as audio or video signals, wirelessly, various telecommunications operations are needed in order to send this data from a transmitting system to a receiving system.

Here the transmitter has the task of preparing the data so that it can be sent via a transmission channel. This transmission channel introduces distortion as well as noise onto the transmit signal. The receiver must then evaluate the received signal in such a way that the transmitted data can be reconstructed again. The figure below illustrates the fundamental relationship.

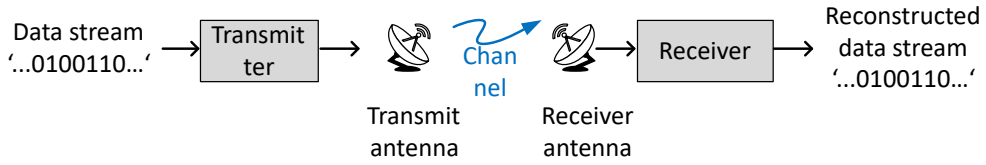

Figure 1-1: Transmitter and receiver for transmitting a binary data stream

The data to be transmitted often exists as a binary data stream (bits). The first step is to convert the bit stream into a symbol stream. This is done by combining multiple bits and referred to as *symbol mapping*. In symbol mapping, the bits are first split up into two processing paths by means of serial-to-parallel conversion. They are referred to as *inphase* (I) and *quadrature (Q) components*.

The values of the symbols can be visualized in so-called I/Q diagrams with their *symbol constellation*. Each symbol constellation has a certain valency *M*. It indicates how many bits (m) per symbol can be transmitted. With an 8-PSK, for example, 3 bits per symbol can be sent:

$m = log_2(M) = log_2(8) = 3$

Figure 1-2 shows typical symbol constellations for satellite communications.

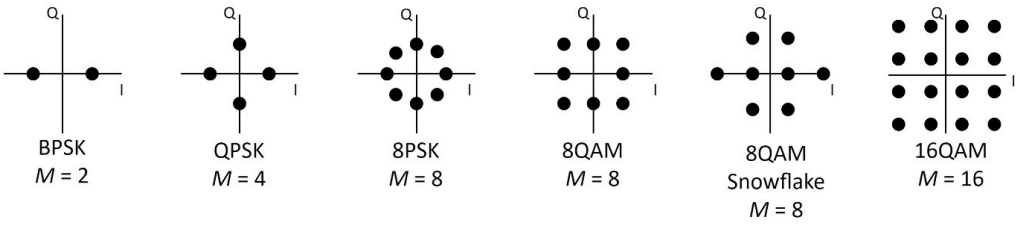

Figure 1-2: Symbol constellations (I/Q diagrams) with their associated valencies M

Here, the baseband signal $x_T(t)$ can be considered as a sequence of symbols a_k, *where* $k \in \mathbb{N}$. The symbol sequences are formed continuously by means of a transmit filter $h_{tx}(t)$ in integer multiples of the symbol clock T_s:

$$x_T(t) = \sum_k a_k \cdot h_{tx}(t - kT_S)$$

Equation 1-1: Transformation of baseband symbols by a transmit filter

In practice, this transmit filter is often designed as a so-called *root-raised-cosine* filter (RRC filter). It can be used to generate a band-limited signal in the frequency range so as not to interfere with adjacent frequency ranges.

Modulation of the two processing paths is performed by a so-called *I/Q modulator*. Figure 1-3 shows the transmitter block diagram for the baseband signal. Here, symbol mapping takes place using a *look-up table* (LUT).

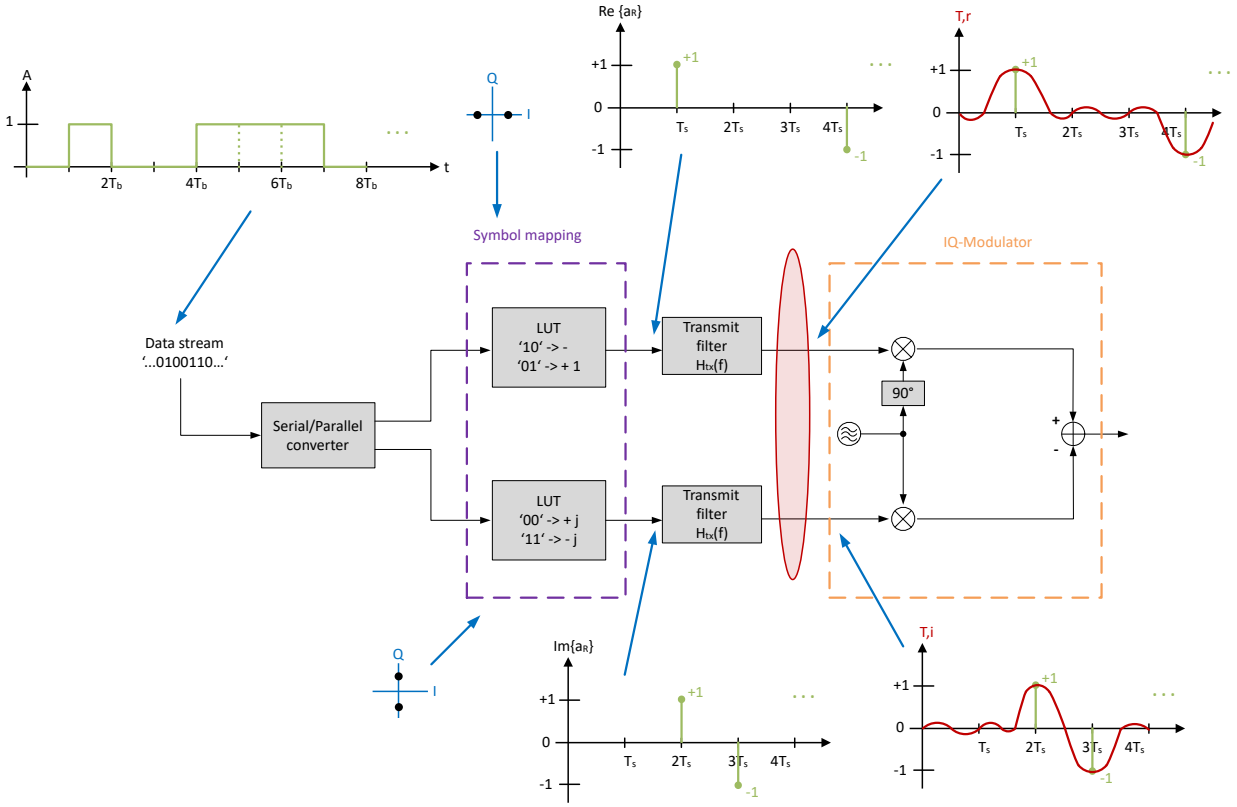

Figure 1-3: I/Q transmitter with a QPSK transmission as an example

Figure 1-4 shows the spectrum of the baseband signal.

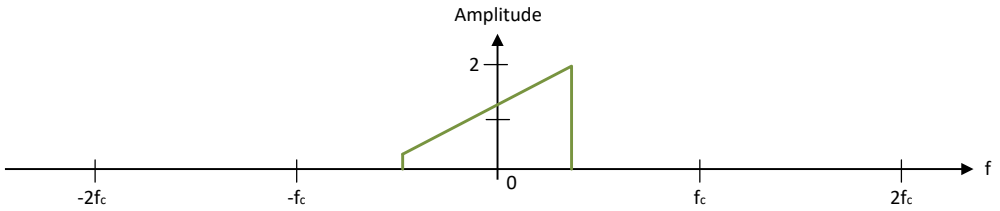

Figure 1-4: Spectrum of baseband signal x_T (t)

In the transmitter, the baseband signal is elevated to the so-called *passband* using the carrier frequency f_c by means of the following complex pointer.

$$e^{j2\pi f_c t}$$

Equation 1-2: Complex pointer for passband transfer

$$\begin{aligned} x_C(t) &= x_T(t) \cdot e^{j2\pi f_c t} = (x_{T,r}(t) + j \cdot x_{T,i}(t)) \cdot e^{j2\pi f_c t} \\ &= (x_{T,r}(t) + j \cdot x_{T,i}(t)) \cdot (\cos(2\pi f_c t) + j \cdot \sin(2\pi f_c t)) \\ &= x_{T,r}(t) \cdot \cos(2\pi f_c t) + j \cdot x_{T,i}(t) \cdot \cos(2\pi f_c t) + j \cdot x_{T,r}(t) \cdot \sin(2\pi f_c t) + \underbrace{j^2}_{-1} \cdot x_{T,i}(t) \cdot \sin(2\pi f_c t) \\ &= (x_{T,r}(t) \cdot \cos(2\pi f_c t) - x_{T,i}(t) \cdot \sin(2\pi f_c t)) + j \cdot (x_{T,i}(t) \cdot \cos(2\pi f_c t) + x_{T,r} \cdot \sin(2\pi f_c t)) \end{aligned}$$

Equation 1-3: Transfer of baseband signal into passband using carrier frequency f_c

Figure 1-5 shows the result in the frequency range.

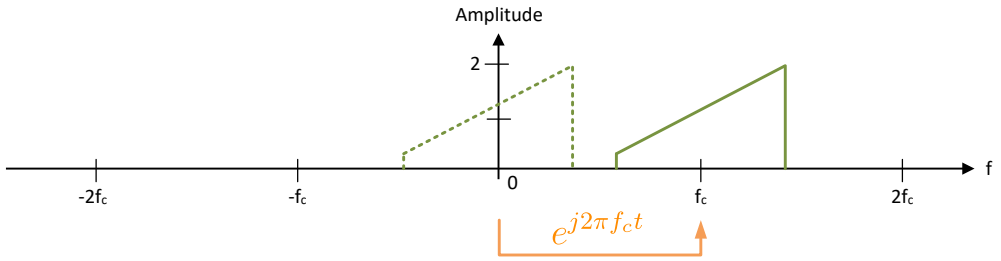

Figure 1-5: Shift of baseband signal into passband by means of carrier f_c

The real part now corresponds to the transmit signal *x(t)* in the passband. The transmit signal can, following amplification, be emitted via an antenna:

$$\begin{aligned} x(t) &= Re\{x_c(t)\} \\ &= Re\{(x_{T,r}(t) \cdot \cos(2\pi f_c t) - x_{T,i}(t) \cdot \sin(2\pi f_c t)) + j(x_{T,i}(t) \cdot \cos(2\pi f_c t) + x_{T,r}(t) \cdot \sin(2\pi f_c t))\} \\ &= x_{T,r}(t) \cdot \cos(2\pi f_c t) - x_{T,i}(t) \cdot \sin(2\pi f_c t) \end{aligned}$$

Equation 1-4: Transmit signal x(t) in passband

Figure 1-6 shows the spectrum of the transmit signal.

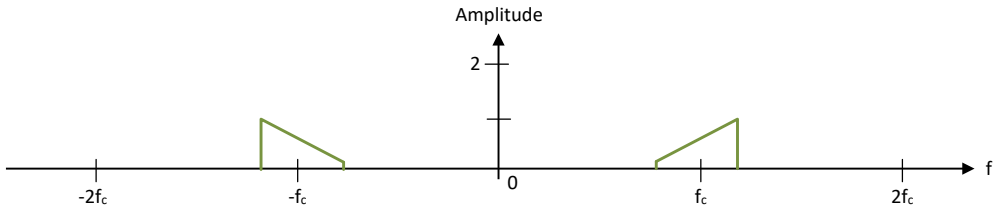

Figure 1-6: Transmit signal x(t) at output of I/Q modulator in transmitter in frequency range

2 Carrier frequency offset

The passband signal *x(t)* passes through the transmit channel. This causes a noise signal *n(t)* to be added to the passband signal. The noise signal should, however, be ignored in the following observations. An *I/Q demodulator* is used for demodulation. It shifts the signal back into the baseband or *zero position*. For this purpose, the signal is multiplied by a complex pointer, i.e.:

$$e^{-j2\pi f_c' t}$$

Equation 2-1: Complex pointer for return to baseband

Unlike the complex pointer of the transmitter, this pointer rotates in the opposite direction.

In practice, however, the carrier frequency f_c of the transmitter does not exactly match the carrier frequency f_c' of the receiver. The frequencies are generated by two different local oscillators (LOs) which owing to their construction and as a result of ambient influences can never have the same frequency. $f_c \neq f_c'$ therefore applies. The receive frequency f_c' deviates from the transmit frequency f_c by a small frequency f_{CFO}. In the case of satellite transmissions in the GHz range, f_{CFO} often lies in the kHz range if the deviation of the local oscillators is in the ppm (*parts per million*) range.

If, e.g. for an 11 GHz carrier frequency (Ku band), the transmit and receive oscillator deviate by 40 ppm from each other, this will result in a CFO of 440 kHz.

This deviation f_{CFO} is referred to as the *carrier frequency offset* and must be corrected in the receiver. The block diagram of the receiver with its processing steps is shown in Figure 2-1 and is explained in the following text.

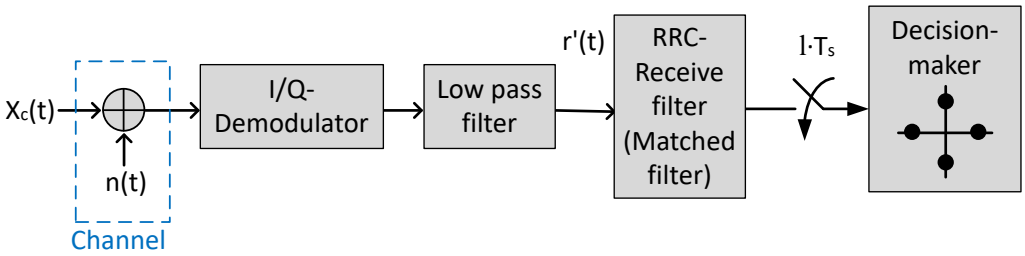

Figure 2-1: Processing stages of receiver

As a result of the demodulation process, the baseband signal *r(t)* now has the following form:

$$r(t) = r_c(t) \cdot e^{-j2\pi f_c' t}$$

Equation 2-2: Baseband signal after demodulation

With:

$$f_c' = f_c + f_{CFO}$$

According to Equation 1-4, this therefore results in:

$$r(t) = (x_{T,r}(t) \cdot \cos(2\pi f_c t) - x_{T,i}(t) \cdot \sin(2\pi f_c t)) \cdot e^{-j2\pi f_c' t}$$

Equation 2-3: Baseband signal after demodulation (2)

If the following substitutions are made:

$$cos(x) = \frac{1}{2}\left(e^{-jx} + e^{jx}\right)$$

$$sin(x) = \frac{j}{2}\left(e^{-jx} - e^{jx}\right)$$

This can be collated as follows:

$$r(t) = \left(x_{T,r}(t) \cdot \frac{1}{2}\left(e^{-j2\pi f_c t} + e^{j2\pi f_c t}\right) - x_{T,i}(t) \cdot \frac{j}{2}\left(e^{-j2\pi f_c t} - e^{j2\pi f_c t}\right)\right).e^{-j2\pi f_c' t}$$
$$= \frac{x_{T,r}(t)}{2}\left(e^{-j2\pi(f_c+f_c')t} + e^{j2\pi(f_c-f_c')t}\right) - j \cdot \frac{x_{T,i}(t)}{2}\left(e^{-j2\pi(f_c+f_c')t} - e^{j2\pi(f_c-f_c')t}\right)$$

Equation 2-4: Baseband signal after demodulation (3)

In the receiver too, and therefore also in the I/Q demodulator, lowpass filtering must be performed in order to eliminate the interfering frequency components $f_c+f_c' \approx 2f_c$. This results in the signal *r'(t)*:

$$r'(t) = \frac{x_{T,r}(t)}{2}e^{j2\pi(f_c-f_c')t} - j \cdot \frac{x_{T,i}(t)}{2}\left(- e^{j2\pi(f_c-f_c')t}\right)$$
$$= \frac{1}{2} \cdot \left(x_{T,r}(r) + j \cdot x_{T,i}(t)\right) \cdot e^{j2\pi(f_c-f_c')t}$$

Equation 2-5: Receive signal after lowpass filtering in I/Q demodulator

Figure 2-2 shows the spectrum of the demodulated signal and Figure 2-3 shows the signal after lowpass filtering.

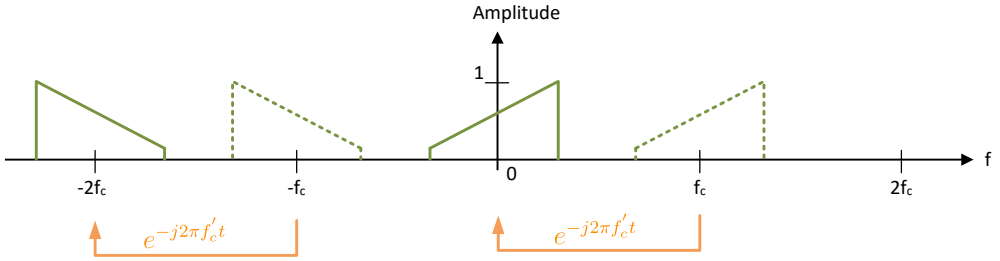

Figure 2-2: Demodulation of receive signal

If the frequency offset f_{CFO} is now used, where $f_c' = f_c + f_{CFO}$, then further transformations can be made:

$$r'(t) = \frac{1}{2} \cdot \left(x_{T,r}(t) + j \cdot x_{T,i}(t) \right) \cdot e^{j2\pi(f_c - (f_c + f_{CFO}))t}$$
$$= \frac{1}{2} \cdot \left(x_{T,r}(t) + j \cdot x_{T,i}(t) \right) \cdot \underbrace{e^{j2\pi(f_c - f_c)t}}_{=e^{j2\pi \cdot 0 \cdot t} = 1} \cdot e^{-j2\pi f_{CFO} t}$$
$$= \frac{1}{2} \cdot \left(x_{T,r}(t) + j \cdot x_{T,i}(t) \right) \cdot e^{-j2\pi f_{CFO} t}$$

Equation 2-6: Demodulated receive signal with frequency offset

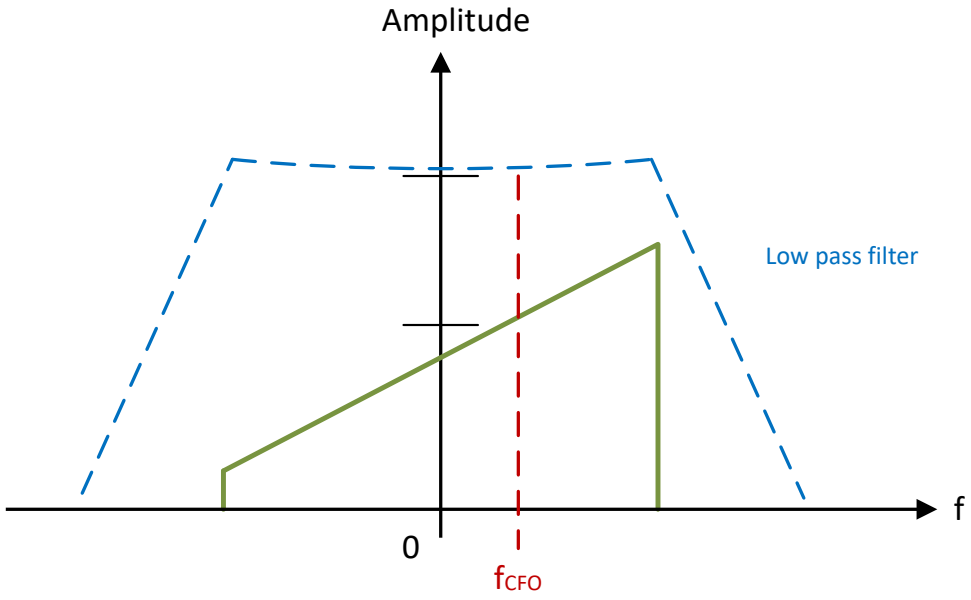

Figure 2-3: Baseband signal after lowpass filtering with residual CFO

From Equation 2-6 it can be seen that, on the one hand, the complex pointer disappears, which indicates optimum demodulation.

$$e^{j2\pi(f_c - f_c)t}$$

Equation 2-7: Complex pointer of I/Q demodulator

However, the CFO with the following pointer still remains:

$$e^{-j2\pi f_{CFO} t}$$

Equation 2-8: Complex pointer of lowpass filtering

The CFO leaves behind a permanent and unwanted shift of the baseband signal toward the zero position in the spectrum, as can be seen in Figure 2-4.

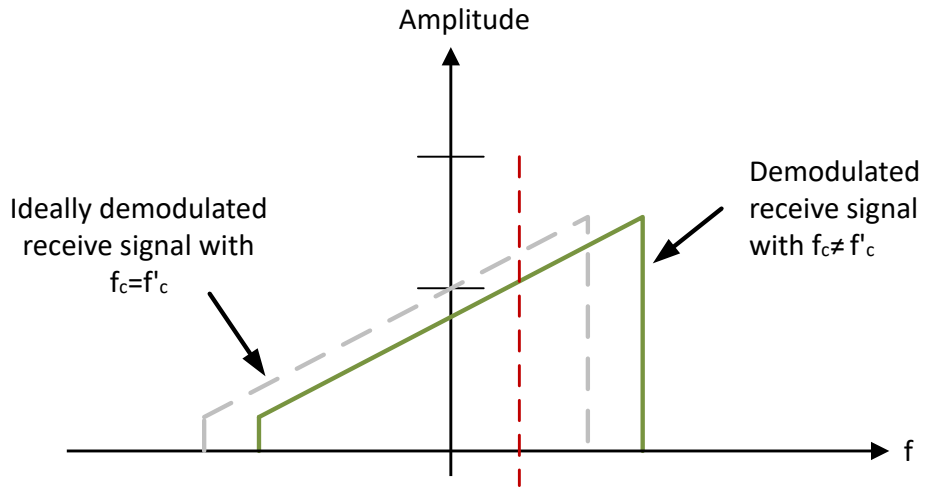

Figure 2-4: Spectrum of baseband signal after demodulation with offset carrier frequency

The receiver must now detect the symbol stream from this signal very effectively again. Detection is possible using a "signal-adapted" receive filter, also referred to as a *matched* filter (MF).

This filter has the same form as the transmit filter $h_{tx}(t)$, but is mirrored in terms of time. $h_{tx}(t) = h_{rx}(-t)$ therefore applies. The advantage of an MF is that it minimizes the influence of noise caused by the transmit channel. In practice, the MF is often implemented as an RRC filter. The output signal of the MF thus produces a *raised-cosine* (RC) signal. The MF performs a convolution operation that outputs the signal $r_{MF}(t)$. The operation is defined by the following integral equation:

$$r_{MF}(t) = x(t) * h(t) = \int x(\lambda) \cdot h_{rx}(t+\lambda) d\lambda$$

Equation 2-9: Convolution integral of MF filter

In this convolution integral we set the baseband signal $r'(t)$, whereby the scaling factor is disregarded. Furthermore, we use the receive filter $h_{rx}(-t)$. This gives:

$$\begin{aligned} r_{MF}(t) &= r'(t) * h_{rx}(-t) \\ &= \int r'(\lambda) \cdot h_{rx}(-t+\lambda) d\lambda = \int r'(\lambda) \cdot h_{rx}(\lambda - t) d\lambda \end{aligned}$$

Equation 2-10: Convolution of baseband signal and receiver filter through MF filter

The symbol sequence formed by the receive filter is used in $r'(\lambda)$:

$$r_{MF}(t) = \int \sum_k a_k \cdot h_{tx}(\lambda - kT_s) \cdot e^{-j2\pi f_{CFO} \cdot \lambda} \cdot h_{rx}(\lambda - t) d\lambda$$

Equation 2-11: Convolution of symbol sequence formed by receive filter, through MF filter

The sum of the symbols a_k is independent of the integration variable λ. This sum can therefore be extracted from the integral. Initially, no further assertions can be made regarding the complex CFO term because this term is dependent on λ:

$$r_{MF}(t) = \sum_k a_k \int h_{tx}(\lambda - kT_s) \cdot e^{-j2\pi f_{CFO}\cdot\lambda} \cdot h_{rx}(\lambda - t)d\lambda$$

Equation 2-12: Convolution of symbol sequence formed by receive filter, through MF filter (2)

By substituting $\vartheta = \lambda - kT_s$, the complex CFO pointer can be split. As a result, a part can be moved in front of the integral:

$$\begin{aligned} r_{MF}(t) &= \sum_k a_k \int h_{tx}(\vartheta) \cdot \underbrace{e^{-j2\pi f_{CFO}\cdot(\vartheta - kT_s)}}_{=e^{-j2\pi f_{CFO}\cdot\vartheta}\cdot e^{+j2\pi f_{CFO}\cdot kT_s}} \cdot \underbrace{h_{rx}(\vartheta + kT_s - t)}_{=h_{rx}(\vartheta-(t-kT_s))} d\vartheta \\ &= \sum_k a_k \cdot e^{+j2\pi f_{CFO}\cdot kT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta - (t - kT_s)) \cdot e^{-j2\pi f_{CFO}\cdot\vartheta} d\vartheta \end{aligned}$$

Equation 2-13: Splitting of complex CFO pointer

At the output of the MF, the still continuous signal $r_{MF}(t)$ is sampled with multiples of the symbol rate t, where $t = lT_s$, $l \in \mathbb{N}$:

$$\begin{aligned} r_{MF}(t = lT_s) &= \sum_k a_k \cdot e^{+j2\pi f_{CFO}\cdot kT_s} \cdot \int h_{tx}(\vartheta) \cdot \underbrace{h_{rx}(\vartheta - (lT_s - kT_s))}_{=h_{rx}(\vartheta-(l-k)\cdot T_s)} \cdot e^{-j2\pi f_{CFO}\cdot\vartheta} d\vartheta \\ &= \sum_k a_k \cdot e^{+j2\pi f_{CFO}\cdot kT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta - (l-k)\cdot T_s) \cdot e^{-j2\pi f_{CFO}\cdot\vartheta} d\vartheta \end{aligned}$$

Equation 2-14: Sampling of output signal of MF

The sum can be split into two partial sums. This demonstrates the problem of subsequent sampling through lT_s. Sampling must be performed such that the sampling times exactly match the symbol times. If this is not the case, i.e. $l \neq k$, then so-called *inter-symbol interference* (ISI) occurs between the received symbols. This means that the symbol sequence $r_{MF}(lT_s)$, which is generated after the MF and sampling, does not match the symbol values of the transmitter. The reason is that the second sum term in Equation 2-15 distorts the consecutive symbols. These distorted symbols may potentially no longer be evaluated correctly and ultimately return an incorrect bit stream.

$$\begin{aligned} r_{MF}(t = lT_s) = &\sum_k a_k \cdot e^{+j2\pi f_{CFO}\cdot kT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta) \cdot e^{-j2\pi f_{CFO}\cdot\vartheta} d\vartheta \\ &+ \underbrace{\sum_{\substack{k \\ l\neq k}}^{l=k} a_k \cdot e^{+j2\pi f_{CFO}\cdot kT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta - (l-k)\cdot T_s) \cdot e^{-j2\pi f_{CFO}\cdot\vartheta} d\vartheta}_{ISI} \end{aligned}$$

Equation 2-15: Splitting into two partial sums with offset sampling times

Figure 2-5 illustrates the situation for the output signal of the MF.

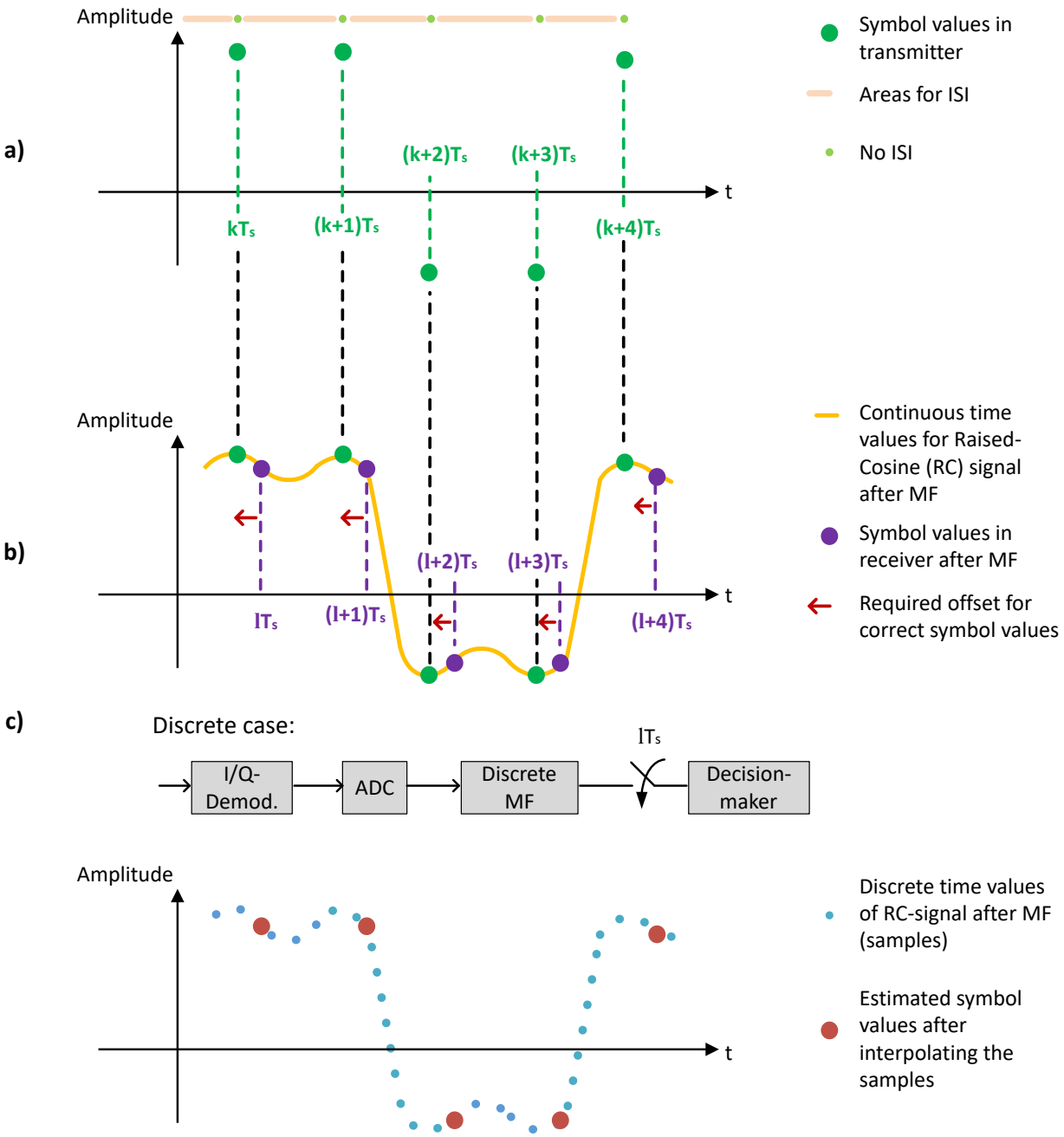

Figure 2-5: Importance of symbol clock synchronization

Figure (a) shows an example of a BPSK symbol stream in the transmitter.

Figure (b) shows the symbol values (violet) in the receiver after the MF. However, these values do not match those in the transmitter. They lie in ranges that give rise to ISI and make symbol clock synchronization necessary.

Figure (c) shows a modern variant of the receiver. The receive signal and MF are digital (SDR (software defined radio) principle). Here too, symbol clock synchronization must be performed to ensure the correct symbol values.

Below we assume ideal sampling through symbol clock synchronization. As a result, the second sum term from Equation 2-15 is not required:

$$r_{MF,i}(t = lT_s) \quad = \sum_l a_l \cdot e^{+j2\pi f_{CFO} \cdot lT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta) \cdot e^{-j2\pi f_{CFO} \cdot \vartheta} d\vartheta$$

Equation 2-16: Receive signal with symbol clock synchronization

Simplification is possible at this point. If the frequency offset f_{CFO} is small enough, the complex pointer of the lowpass filtering may cause only a negligibly small change to $h_{rx}(\vartheta)$ across the integration period.

$$e^{-j2\pi f_{CFO} \cdot \vartheta}$$

Equation 2-17: Complex pointer of lowpass filtering

$h_{rx}(\vartheta)$ can therefore be assumed. This results in the equation:

$$r_{MF,i}(t = lT_s) = \sum_l a_l \cdot e^{+j2\pi f_{CFO} \cdot lT_s} \cdot \int h_{tx}(\vartheta) \cdot h_{rx}(\vartheta) d\vartheta$$

Equation 2-18: Receive signal with sufficiently small frequency offset

The integral is referred to as the *autocorrelation function* (ACF). It performs a convolution of the filter function on itself. Here, the integral can assume the following values:

$$x_i((l-k)T_s) = \int h_{tx}(\vartheta) \cdot (\vartheta - (l-k) \cdot T_s) d\vartheta = \begin{cases} 1 \text{ für } l-k=0 \\ 0 \text{ für } l-k \neq 0 \end{cases}$$

Equation 2-19: Autocorrelation function

The symbol sequence originally sent is now created in the receiver. It consists of the (desired) symbol stream a_l and a complex pointer. The symbol sequence has the following structure:

$$r_{MF,i}(lT_s) = \sum_l a_l \cdot e^{+j2\pi f_{CFO} \cdot lT_s}$$

Equation 2-20: Received symbol sequence

Figure 2-6 a) shows the effects of $r_{MF,i}(kT_s)$ in the I/Q diagram. It is also evident that an uncorrected CFO in the receiver can lead to errors in the downstream detector. The symbol values are rotated progressively in terms of time as a result of the complex pointer:

$$e^{+j2\pi f_{CFO} \cdot lT_s}$$

Equation 2-21: Time rotation of symbol values in detector

In the detector, the decision thresholds for the symbol values are exceeded as a result.

Figure 2-6 b) shows the typical effect caused by rotation of the symbols. It was assumed here that the frequency offset f_{CFO} is positive. That is, the receiver has a slightly higher receive frequency than the transmitter. If the opposite is the case, i.e. if transmission takes place at a slightly lower frequency, the symbols in the diagram would rotate in the opposite direction.

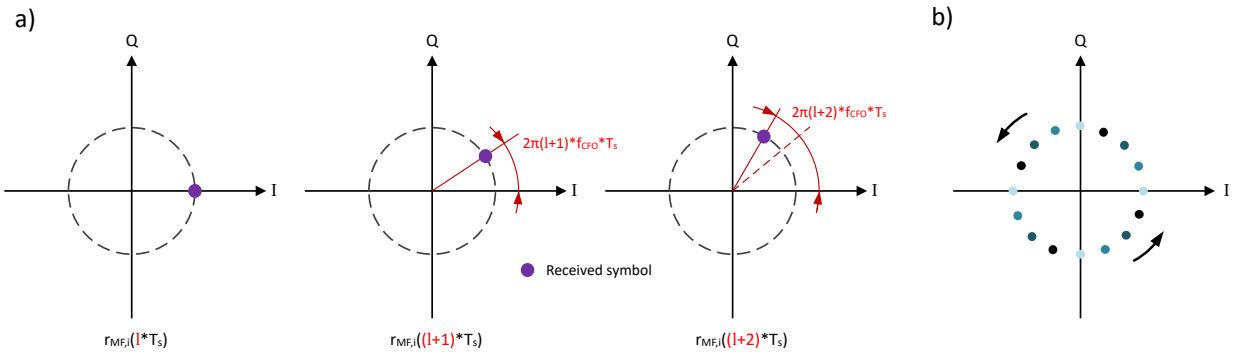

Figure 2-6: Effects of CFO on symbol constellation

Figure a): The received symbols are rotated at three discrete times and identical symbol values (here "+1").

Figure b): If the CFO is not eliminated, this is evident as rotating symbol values in the I/Q diagram of the receiver. In the subsequent detector stage, this leads to incorrectly estimated symbol values.

3 Carrier bandwidths

The bandwidth characterizes the part of the frequency spectrum in which the dominant frequency components of a signal to be transmitted or stored are located.

Ideally, there is no signal energy outside of this bandwidth. However, each transmit signal is limited in terms of time and its bandwidth is thus infinite. It is therefore more important for the signal energy of the transmit signal outside of certain frequencies to be so low that adjacent signals in the frequency range are not disturbed.

For the bandwidth of passband signals, the carrier frequency f_c is used as a reference point. Bandwidths are commonly specified together with a dB value, e.g. the *3-dB bandwidth* or *10-dB bandwidth*; see also Figure 3-1. Here, the signal maximum in the spectrum is assumed to be at f_c. From the signal maximum, drop down by 3 dB and 10 dB. The intersect points of these threshold values with the signal give the lower and upper *limit frequencies* $f_{gu,3dB}$/$f_{go,3dB}$ and $f_{gu,10dB}$/$f_{go,10dB}$. The associated bandwidths are then calculated as follows:

$$B_{3dB} = f_{go,3dB} - f_{gu,3dB}$$

Equation 3-1: 3 dB bandwidth

$$B_{10dB} = f_{go,10dB} - f_{gu,10dB}$$

Equation 3-2: 10 dB bandwidth

The 3-dB bandwidth is often referred to as the *half-power bandwidth* because here the power density is reduced by half compared to its maximum value. We have:

$$10 \cdot \log\left(\frac{P_{ref}}{P}\right) = 10 \cdot \log_{10}(0.5) = -\frac{1}{\sqrt{2}}\ dB = -3.01\ dB \approx -3dB$$

Equation 3-3: Half-power bandwidth

However, signals can also occur whose maximum values are not located at the center of the spectrum owing to a discrete spectral line or whose signal spectrum is even asymmetrical. A further bandwidth definition uses only the area of the signal spectrum. It integrates the *power spectral density* (PSD) over a range *S(f)*. The power spectral density indicates how the signal power is spread across the frequency. A power is therefore calculated for the bandwidth definition:

$$\int_{f_{gu,\alpha}}^{f_{go,\alpha}} S(f)df = \alpha \int_{-\infty}^{\infty} S(f)df$$

Equation 3-4: Calculation of power spectral density

Here α is a percentage setpoint for the area of the signal in the spectrum. For example, the *99% bandwidth* with α = 0.99 describes the frequency range of the signal in which 99% of the signal power is located. (Signal power = area below the signal in the frequency range.) 0.5% signal power thus remains in both the lower and upper frequency range, as shown in Figure 3-1. The *99% bandwidth* is sometimes also referred to as the *occupied bandwidth*.

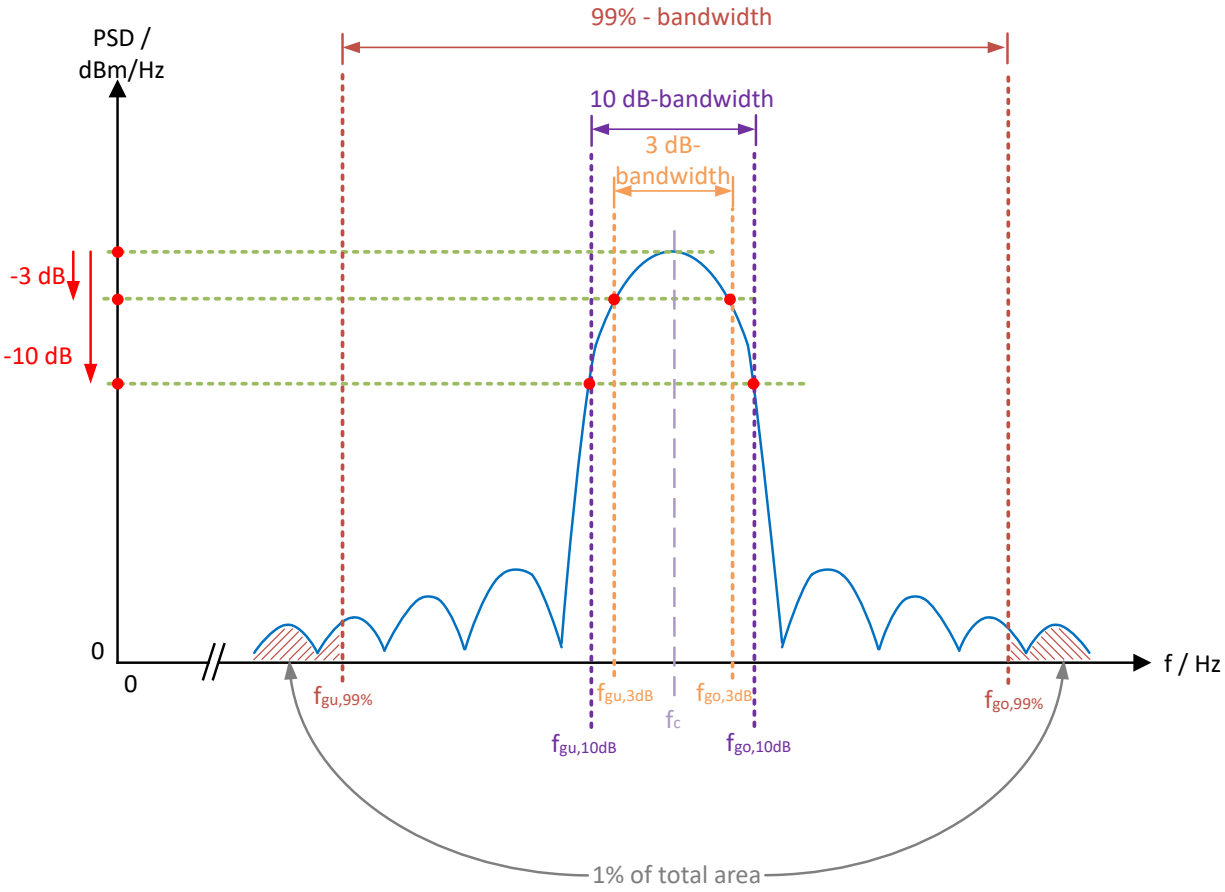

Figure 3-1: Example representation of various bandwidth definitions

For RC filters, the symbol rate T_s and *roll-off factor* α are required. The roll-off factor influences the slope steepness of the signal in the frequency range. The bandwidth of the passband signal can then be calculated as:

$$B_{RC} = \frac{1+\alpha}{T_s} \text{ mit } 0 \leq \alpha \leq 1$$

Equation 3-5: Bandwidth of passband signal

The associated spectrum of the passband signal is shown in Figure 3-2. B_N corresponds to half the bandwidth of the passband signal.

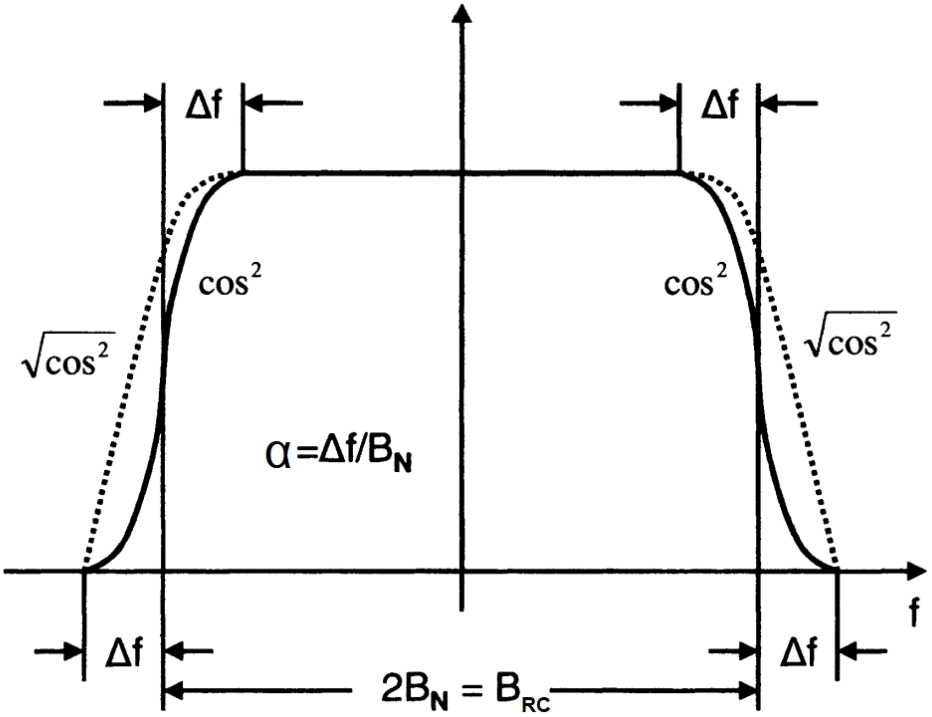

Figure 3-2: Spectrum of an RRC and RC signal as a function of roll-off factor α

The RRC filter is also known as the root-cosine filter ($\sqrt{cos^2}$).

4 Carrier power C

Carrier power C describes the power downstream of the transmit antenna up to the receiver input.

It is influenced by free-space attenuation l_{FS} and other combined losses l_0.

$$l_{FS} = \left(\frac{4\pi r}{\lambda}\right)^2 = \left(\frac{4\pi r f}{c}\right)^2$$

Equation 4-1: Free-space attenuation

The other losses l_0 include attenuation caused by rain, clouds, the atmosphere, scintillation losses, receiver components as well as a potential alignment and polarization error of the transmit and receive antenna.

Scintillation losses are caused by a variation in the refractive index between troposphere and ionosphere. Here, the receive power downstream of the receive antenna is:

$$\begin{aligned} C = P_r &= P_t \cdot G_t \cdot G_r \cdot \frac{1}{l_{FS} \cdot l_0} \\ &= EIRP \cdot G_r \cdot \frac{1}{l_{FS} \cdot l_0} \end{aligned}$$

Equation 4-2: Receive power downstream of receive antenna (Friis transmission equation)

Where:

P_t: transmit power

G_t: gain of the transmit antenna

G_r: gain of the receive antenna

This equation is also known in communications engineering as the *Friis transmission equation*.

The EIRP (*equivalent isotropic radiated power*) value can be expressed as EIRP = $P_t \cdot G_r$. If the carrier power is specified as a dB value, the logarithmic laws can be applied. In other words, multiplication is replaced by addition, i.e. $\log(a \cdot b) = \log(a) + \log(b)$. Division is replaced by subtraction, i.e. $\log(a/b) = \log(a) - \log(b)$. Where $a,b \in \mathbb{R}$ and $a,b > 0$ respectively. Hence, the following applies:

$$C[dB] = P_t[dBW] + G_t[dBi] + G_r[dBi] - (10 \cdot \log_{10}(l_{FS}) + 10 \cdot \log_{10}(l_0))[dB]$$

Equation 4-3: Logarithmic EIRP value

The carrier power can also be expressed as a function of bit energy E_b as well as bit time T_b and bit rate R_b.

$$C\ [W] = \frac{E_b}{T_b} = E_b \cdot Rb$$

Equation 4-4: Carrier power as a function of bit energy

5 Noise power spectral density N_0 and noise power N

Noise refers to nondeterministic signal fluctuations which can be described as a statistical process. Here, the noise power is added to the carrier power and thus complicates demodulation in the receiver. For satellite transmissions, the effect of noise must be taken into consideration because noise occurs at many points in the transmission system and influences signal quality.

The sources of noise are:

- Natural sources in the area surrounding the receive antenna (ground, rain, atmosphere (static discharge) as well as cosmic noise)
- System components of the receiver (receive antenna, amplifier stages (LNA (low-noise amplifier), IF (intermediate frequency) amplifier), connecting cables, mixer). The largest noise contributions are usually produced by the receive antenna and the LNA. The gain level of the LNA influences the noise contributions of the downstream stages.

Unwanted signals in the receiver are referred to as *interference*. The principal cause of noise in the receiver (up to 100 GHz) is *thermal noise*. This occurs as a result of the random movements of electrons. The main parameter governing this movement is the *system noise temperature T* in Kelvin (0 °C≙273 K). In this case, the *noise power density* N_0 is calculated as follows:

$$N_0[\frac{W}{Hz}] = k \cdot T$$

Equation 5-1: Noise power density N_0

Where k is a proportionality factor, the *Boltzmann constant*:

$$k = 1.38 \cdot 10^{-23} \frac{J}{K} = 1.38 \cdot 10^{-23} \frac{W}{Hz \cdot K}$$

Equation 5-2: Boltzmann constant k

T =290 *K* (≙17 °*C*) is often selected as the noise temperature. $N_0 = 4 \cdot 10^{-21}$ *W/Hz* thus takes on a manageable value. In the logarithmic scale, the noise power density can also be calculated as follows:

$$N_0[dBW/Hz] \quad = K \left[dB \cdot \frac{Ws}{K}\right] + 10 \cdot \log_{10}(T)[dBK]$$

Equation 5-3: Noise power density N_0 (in dB)

With the Boltzmann constant in logarithmic form:

$$k = -228.6\ dB \cdot \frac{J}{K} = -228.6\, dB \cdot 10^{-23} \frac{W_s}{K}$$

Equation 5-4: Boltzmann constant k (in dB)

The *noise power N* at the receiver input, which influences the signal, depends on the (noise) bandwidth *B* which is allowed to pass through. Here, B ≈ 1 to 1.5 * 3-dB bandwidth.

$$N\ [W] = k \cdot T \cdot B$$

Equation 5-5: Noise power N at receiver input

In the logarithmic scale:

$$N\ [dBW] \quad = K\ \left[dB\frac{W_s}{K}\right] + 10 \cdot \log_{10}(T)[dBK] + 10 \cdot \log_{10}(B)[dBHz]$$

Equation 5-6: Noise power N at receiver input (in dB)

The *noise bandwidth B* is the bandwidth that contains the information-bearing signal. From Equation 5-6, it can be seen that the noise power is constant over frequency. This is why it is sometimes colloquially referred to as the "noise floor".

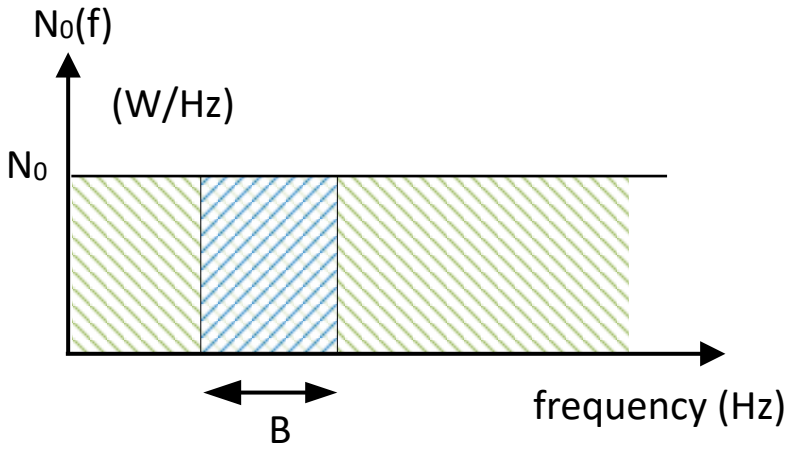

Figure 5-1: Noise power density of white, Gaussian noise in frequency range

This kind of noise is also referred to as *white noise*. In the time domain, it has a zero-mean Gaussian distribution (*additive white Gaussian noise* (AWGN)).

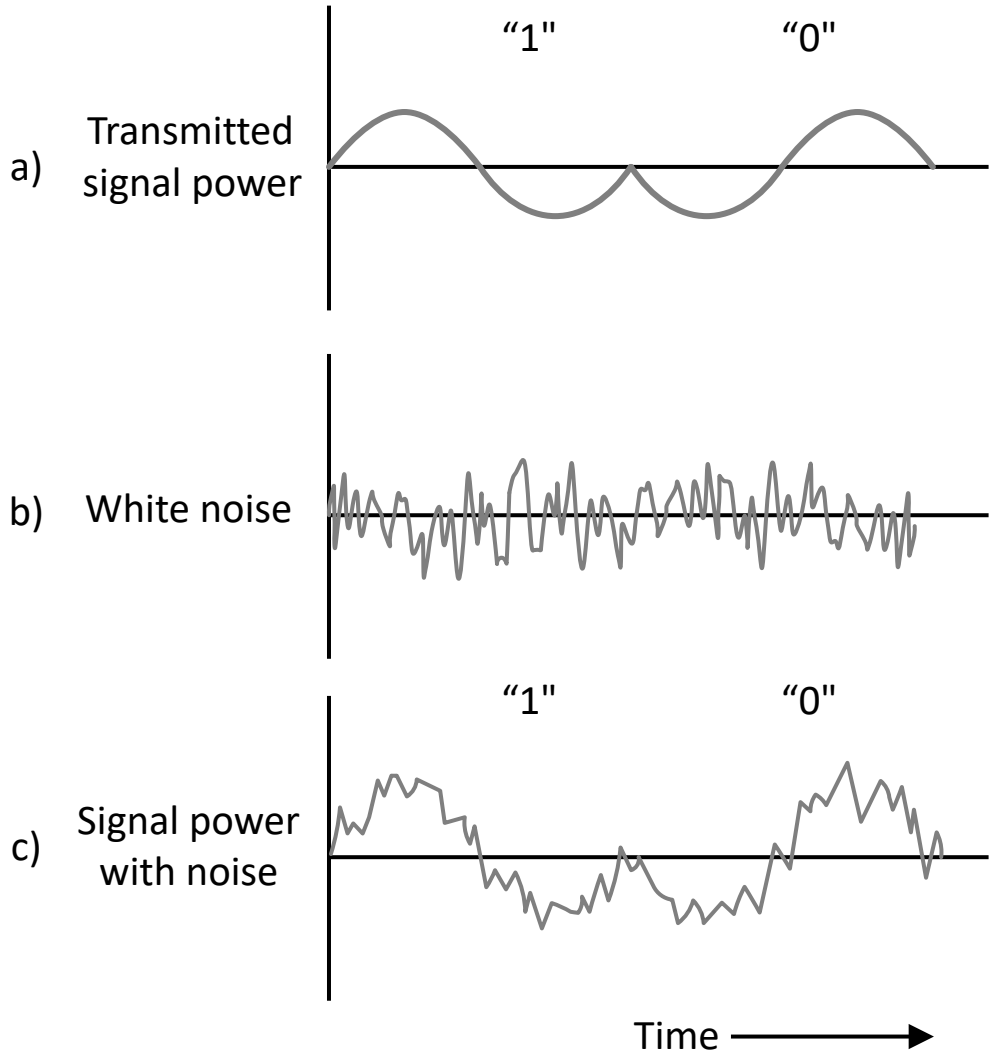

Figure 5-2: Example of BPSK signal with accumulated white noise

The noise power density N_0 can therefore also be regarded as a noise power that is normalized to a 1-Hz bandwidth. Because for the white noise the noise power is independent of the frequency and is constant, the value of the noise power density N_0 in *W/Hz* is sometimes also used:

$$N_0 = \frac{N}{B} = \frac{k \cdot T \cdot B}{B}$$

Equation 5-7: Noise power density for white noise

From Equation 4-4, transposition results in $E_b = C*T_b$. Using Equation 5-7, it is now possible to extrapolate the variable E_b/N_0, i.e. *bit energy relative to noise power density*. This variable is sometimes also referred to as the "signal-to-noise ratio per bit (SNR per bit)":

$$\frac{E_b}{N_0} = \frac{C \cdot T_b}{N/B} = \frac{C}{N} \cdot T_b \cdot B = \frac{C}{N} \cdot \underbrace{\frac{B}{R_b}}_{1/\varrho}$$

Equation 5-8: Bit energy relative to noise power density (SNR per bit)

In the logarithmic scale:

$$\frac{E_b}{N_0} = \frac{C}{N}[dB] + 10 \cdot \log_{10}(B) - 10 \cdot \log_{10}(R_b)$$

Equation 5-9: Bit energy relative to noise power density (in dB)

The factor *C/N* is called the *carrier-to-noise value*. Equation 5-9 shows that E_b/N_0 and *C/N* differ only by one factor. The factor is determined by the bandwidth and bit rate and corresponds to the inverse of the so-called *spectral efficiency* ϱ. This specifies how many "bits per second" (bit rate) can be transmitted on a certain bandwidth. If the bandwidth is the same as the bit rate, E_b/N_0 and *C/N* are identical.

In addition to the E_b/N_0 value, there is also the E_s/N_0 value, which describes the *symbol energy relative to the noise power density*. Both values are linked to each other by the symbol valency *M*:

$$\frac{E_s}{N_0} = \frac{E_b}{N_0} \cdot \log_2(M)$$

Equation 5-10: Symbol energy relative to noise power density

In the logarithmic scale:

$$\frac{E_s}{N_0} = \frac{E_b}{N_0}[dB] + \log_2(M)$$

Equation 5-11: Symbol energy relative to noise power density (in dB)

The value E_b/N_0 is a key factor in digital signal transmission. It influences the (bit) error probability P_b ($P_b \sim E_b/N_0$) and is directly proportional to the carrier power *C*. The value serves as a starting point for planning satellite links. The error probability is lower, the greater E_b/N_0 is. From E_b/N_0, it is possible to determine the bit error rate (BER). To do so, C/N must be converted to E_b/N_0. Figure 5-3 shows the BER over E_b/N_0. Here the symbol constellations from Figure 1-2 with their symbol valencies *M* have different curves.

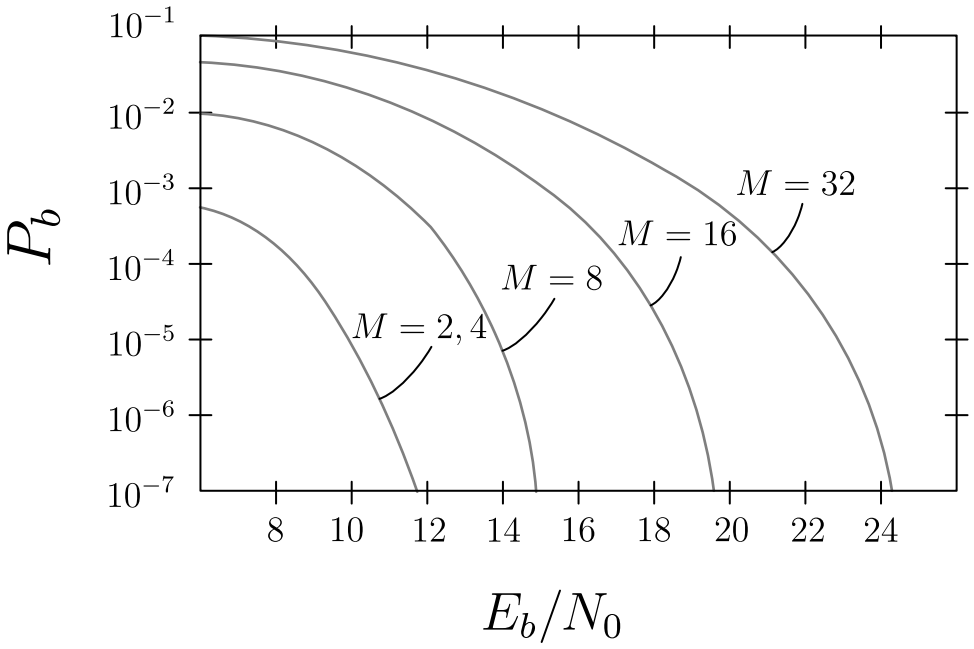

Figure 5-3: Bit error probability P_b (BER = bit error rate) for M-PSK with AWGN noise

The curves in Figure 5-3 describe the achievable performance of a digital transmission.

To reduce the error probability, the value *C* or *B* must be increased while maintaining a constant data rate R_b. In practice, however, this is not always possible. This is because the satellite system is restricted in terms of its power and bandwidth, and regulatory requirements also limit this. Error correction processes can, however, reduce the error probability P_b in the receiver.

If Equation 5-8 is converted to *C/N*, this results in:

$$\frac{C}{N} = \frac{E_b}{N_0} \cdot \frac{R_b}{B}$$

Equation 5-12: Carrier-to-noise

And for the C/N_0 parameter:

$$\frac{C}{N_0} = \frac{E_b}{N_0} \cdot R_b$$

Equation 5-13: Carrier-to-noise (normalized)

If the bit rate of the receive signal is known, the ratio E_b/N_0 can be determined from the C/N and C/N_0 parameters explained below. This additionally depends on, among other things, the modulation type, channel coding and receiver synchronization. The greater the C/N, C/N_0 or E_b/N_0 value is, the better the *system performance* will be.

6 Carrier-to-noise ratio C/N

With a received signal, its absolute receive level is not the important factor. What is important is that the receiver is able to separate this signal sufficiently well from the added noise level and can thus demodulate the signal. With satellite links in particular, it may additionally be the case during transmission that the power of the receive signal is only in the picowatt range.

If such a weak signal is received, gain in the receiver will not only amplify this signal, it will also amplify its associated noise. The quality of the receive signal is therefore determined by a ratio of signal power *C* of the carrier to noise power *N* at the input of the receiver. This ratio is referred to as the *carrier-to-noise*(C/N, CNR) value.

Here, the C/N value is an important parameter in characterizing the performance of a transmission system. The transmission quality is better, the greater the C/N value is. It is given in dB. A typical lower receive limit would be a value from 6 dB to 10 dB. If frequency spread processes are applied, this limit can even assume negative values. For satellite links, multiple C/N values exist. We make a distinction between values for the link from the ground station to the satellite (uplink, $(C/N)_U$). And also between values for the link from the satellite to the ground station (downlink, $(C/N)_D$).

For a connecting link, the noise power *N* at the receiver input is:

$$N[W] = k \cdot T \cdot B$$

Equation 6-1: Noise power at receiver input for a simple satellite link

The C/N value is the result of the transmit power, antenna gains and attenuation losses that occur on the transmission path. Using Equation 4-2, the value can therefore be set up as a ratio equation for carrier power and noise power:

$$\frac{C}{N} = \frac{P_r}{N} = \frac{P_t \cdot G_t \cdot G_r \cdot \frac{1}{l_{FS} \cdot l_0}}{k \cdot T \cdot B}$$

$$= \frac{EIRP}{k \cdot B} \cdot \frac{G_r}{T} \cdot \frac{1}{l_{FS} \cdot l_0}$$

Equation 6-2: C/N value as a function of transmission performance

We see that the smaller the carrier power is or the greater the noise power is, the greater the reduction in performance will be. The expression can thus be given in the logarithmic scale as:

$$\frac{C}{N}[dB] = EIRP[dBW] + K\left[db\frac{Ws}{K}\right] + \underbrace{G_r[dBi] - 10 \cdot \log_{10}(T)[dBK]}_{=:\frac{G}{T}} - (L_{FS} + L_0)[dB]$$

$$- 10 \cdot \log_{10}(B)[dBHz]$$

Equation 6-3: C/N value as a function of transmission performance (in dB)

The ratio of receive antenna gain to system noise temperature, *G/T* in dB/K, is an important metric *(figure of merit)* for the performance of the receiver.

The C/N value can be determined by reading level values from the spectrum. Figure 6-1 shows an example spectrum. The important thing is that during this measurement the noise power *N* consists of *multiple noise components*, as explained in Chapter 8, "Total C/N", on page 27.

With the selected bandwidth *B*, the noise power density N_0 allows the noise power $N = N_0 \cdot B$ into the receiver. The signal that is within the bandwidth *B* is composed of the carrier power and noise power (i.e. *C+N*). If the C/N value is determined using a spectrum analyzer, it is important to note that the noise is also within the carrier bandwidth. The spectrum analyzer therefore shows the sum of noise power and carrier power in the spectrum. Using the following equation, it is possible to derive the C/N ratio from the two read values *C + N* and *N*.

$$\frac{C+N}{N} = \frac{C}{N} + \frac{N}{N} = \frac{C}{N} + 1$$

Equation 6-4: Determining C/N value using a spectrum analyzer

The sought C/N ratio is obtained by transposition as follows:

$$\frac{C}{N} = \frac{C+N}{N} - 1$$

Equation 6-5: Determining C/N value using a spectrum analyzer (2)

The value in Equation 6-5 is given as a power ratio and not as a dB value.

The following applies across the bandwidth under consideration:

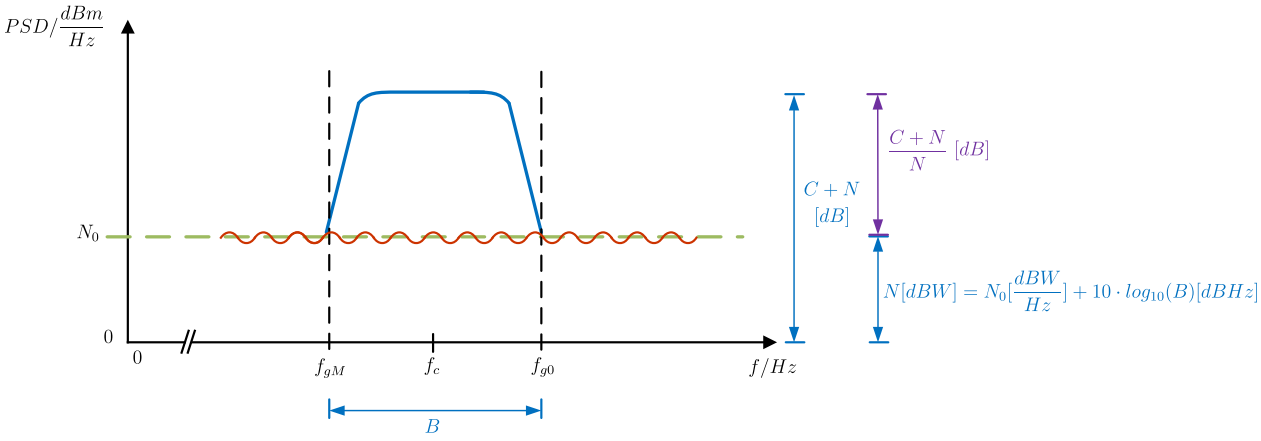

Figure 6-1: Determining C/N ratio using a spectrum analyzer

The C/N ratio can be determined by using a spectrum analyzer to identify the level values (C+N) and N within the bandwidth B.

To calculate in the logarithmic scale, the value for (C+N)/N must first be read from the spectrum in dB. This value is converted into a linear ratio. "1" is then subtracted from this. The result is converted back into decibels.

If, for example, the (C+N)/N ratio is 12 dB, then its linear ratio is $10^{12/10}$ = 15.84. The true (linear) C/N ratio is 14.84, i.e. 10 $\log_{10}$(14.84) = 11.71 dB.

For large (C+N)/N ratios in the spectrum (> 10 dB), *(C + N)/N ≈ C/N* can be set as a rough, initial estimate. Using a spectrum analyzer, the C/N ratio of the signal spectrum is in practice determined by means of area integration over the power spectral density *S(f)*. With sufficiently high (C+N)/N ratios and a sufficiently low noise power *N*, the C/N ratio can thus be estimated as follows:

$$\frac{C}{N}[dB] \approx 10 \cdot \log_{10}\left(\int_{f_{gu}}^{f_{go}} S(f)df\right)$$

Equation 6-6: Determining C/N value with sufficiently low noise power

7 Carrier-to-noise density ratio C/N_0

The C/N and C/N_0 parameters are linked to each other across the bandwidth *B*. This means that, in addition to the C/N value, the C/N_0 value is also used in the planning and design of satellite links. The C/N_0 value can be calculated using the following equation:

$$\frac{C}{N} = \frac{C}{N_0} \cdot \frac{1}{B}$$

Equation 7-1: C/N value as a function of bandwidth

or

$$\frac{C}{N_0} = \frac{C}{N} \cdot B$$

Equation 7-2: C/N value (normalized) as a function of bandwidth

Or as a dB value, this results in the following:

$$\frac{C}{N_0}[dB] = \frac{C}{N}[dB] + 10 \cdot \log_{10}(B)[\mathrm{dBHz}]$$

Equation 7-3: C/N value (normalized, in dB) as a function of bandwidth

Here too, the higher C/N_0 is, the better the transmission quality will be. As the C/N_0 values include the bandwidth of the signal, they are, however, often much higher than the C/N values.

8 Total C/N

Apart from the effects of the noise in the uplink and downlink, N_U and N_D, *interference*, N_I and *intermodulation products*, N_{IM} must also be taken into consideration.

This results in the total C/N expressed as:

$$\frac{C}{N} = \frac{(C + N_U + N_D + N_I + N_{IM})}{(N_U + N_D + N_I + N_{IM})}$$

Equation 8-1: Total C/N with interference and intermodulation products

Interference is caused by unwanted power components of signals in the spectrum. These power components come from e.g. other satellite links and are within the bandwidth to be demodulated. Figure 8-1 illustrates the problem.

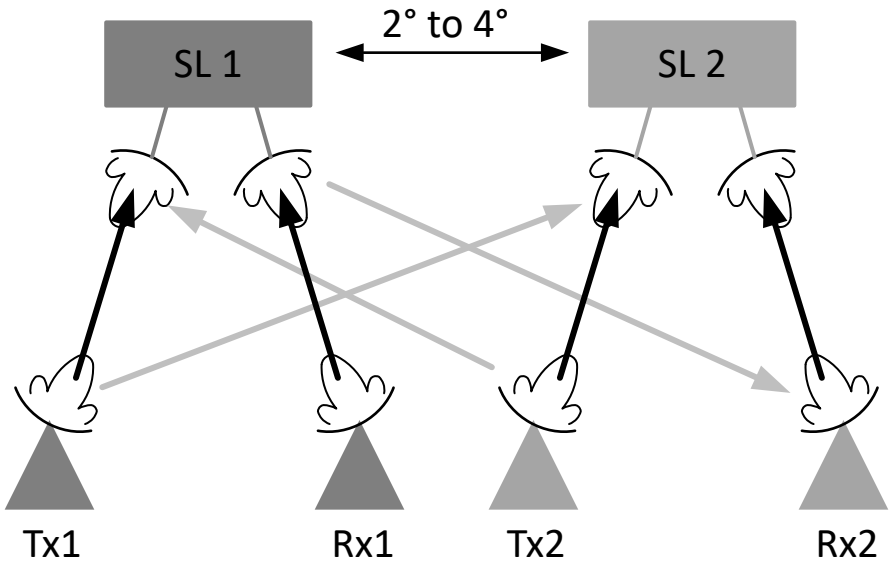

Figure 8-1: Interference occurring during reception, caused by other ground stations or satellites

High-power amplifiers (HPA) such as *traveling wave tube amplifiers* (TWTA) and *solid-state power amplifiers* (SSPA) do not possess nonlinear characteristics. If signals are amplified by such HPAs, this can give rise to intermodulation products. The intermodulation products are power components in the spectrum that are evident on harmonics (integer components) of the carrier frequency. Figure 8-2 shows the qualitative intermodulation products for the amplification of two carrier frequencies.

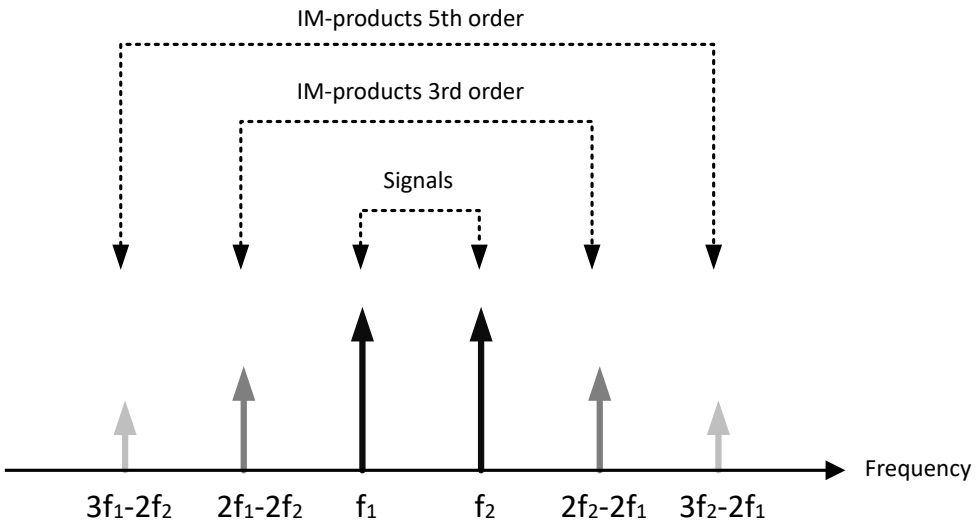

Figure 8-2: Diagram showing harmonics of intermodulation products of two amplified carriers in spectrum

It may therefore be the case that power components of the interference *I* and intermodulation products *IM* also occur in addition to the noise power *N*. The effects of these components can be considered as an increase in the (thermal) noise in the satellite link, both in the uplink and downlink. The resulting noise power N_{tot} is formed by adding together its power components as follows:

$$N_{ges} = N_D + N_U + N_I + N_{IM}$$

Equation 8-2: Total noise power in overall satellite link

This noise power is seen when determining the total C/N ratio, as already shown in the example in Figure 6-1. This measurement of the (C+N)/N ratio therefore corresponds in full form to:

$$\frac{C+N}{N} = \frac{C+N_{ges}}{N_{ges}} = \frac{C+N_D+N_U+N_I+N_{IM}}{N_D+N_U+N_I+N_{IM}}$$

Equation 8-3: Total C/N for overall satellite link

This value can be adopted as the C/N value for sufficiently high dB values, as described in Equation 6-6.

The effects of all noise components can be seen alongside further additional influences (e.g. the CFO) in the I/Q diagram. Following matched filtering in the receiver, they lead to deviation of the received symbols from their intended, ideal point. Figure 10-1 shows this relationship in the I/Q diagram.

9 Signal-to-noise ratio S/N

Following matched filtering using the RRC filter, the C/N ratio is referred to as the *signal-to-noise ratio* (SNR) of the symbols. In other words, the C/N ratio is measured before the demodulation and the signal-to-noise ratio after the demodulation.

The starting point is the equation for calculating the symbol energy relative to the noise power density:

$$\frac{E_s}{N_0} = \frac{S \cdot T_s}{N/B} = \frac{S}{N} \cdot T_s \cdot B$$

Equation 9-1: Calculation of symbol energy relative to noise power density

S corresponds to signal power. Here, like the noise power *N*, the noise power density N_0 is composed of the individual influencing components:

$$N_0 = N_{0,D} + N_{0,U} + N_{0,I} + N_{0,IM}$$

Equation 9-2: Total noise power density

The bandwidth is substituted by the sample rate T_{sample} of the system, i.e. $B=1/T_{sample}$. Transposition results in the SNR *S/N* as follows:

$$\frac{S}{N} = \frac{E_s}{N_0} \cdot \frac{1}{T_s \cdot B} = \frac{E_s}{N_0} \cdot \frac{T_{sample}}{T_s}$$

Equation 9-3: SNR as a function of sample rate

In the logarithmic scale:

$$\frac{S}{N}[dB] = \frac{E_s}{N_0}[dB] + 10 \cdot \log_{10}\left(\frac{T_{sample}}{T_s}\right)$$

Equation 9-4: SNR as a function of sample rate (in dB)

The sample rate T_{sample} is matched to the symbol clock T_s. The SNR *S/N* then corresponds to the symbol energy relative to the noise power density E_s/N_0:

$$\frac{S}{N}[dB] = \frac{E_s}{N_0}[dB] + 10 \cdot \underbrace{\log_{10}(1)}_{=0} = \frac{E_s}{N_0}[dB]$$

Equation 9-5: SNR with matched sample rate

10 Error vector magnitude EVM

From the total quantity *L* of symbols, a quantity *J* of symbols is selected for the measurement.

Using the I/Q diagram, three vectors can be calculated from this quantity for each symbol index j; see Figure 10-1:

- Vector $d_{j,ideal}$ for the ideal position of the symbol
- Detection vector $d_{j,signal}$ for the received symbol (plotted in red in Figure 10-1)
- Error vector $d_{j,error} = d_{j,ideal} - d_{j,signal}$, which occurs as a result of the interference and distortion during transmission

The smaller the lengths determined for the error vectors are, the smaller the applied interfering factors of the symbols will be and the better the receive quality will be.

Here, the *error vector magnitude* (EVM) gives the error vector as a percentage. The EVM normalizes the mean power of the error vector to the mean power of all symbols (of the same valency *M*) from the quantity L:

$$EVM[\%] = \sqrt{\frac{P_{error}}{P_{ref}}} \cdot 100\% = \sqrt{\frac{\frac{1}{j}\sum_j |d_{j,ideal} - d_{j,signal}|^2}{\frac{1}{L}\sum_l |d_{l,signal}|^2}} \cdot 100\%$$

Equation 10-1: Calculation of EVM

Where

- P_{error}: mean power of the error vector
- P_{ref}: mean power of all symbols

The ideal positions $d_{j,ideal}$ can be estimated by the receiver (*non-data-aided estimation*). Or they are known to the receiver through training sequences in the symbol stream (*data-aided estimation*). Another definition of the EVM does not normalize using the mean power P_0, but instead using the peak power of the symbols. With a 16-QAM, for example, this leads to different results. For this reason, precise definition is essential when comparing EVM values that were obtained using different measurement systems.

For a sufficiently large quantity of symbols *L*, the SNR in the baseband can finally be estimated from the EVM:

$$\frac{S}{N} \approx \frac{1}{\left(\frac{EVM[\%]}{100}\right)^2}$$

Equation 10-2: Estimation of SNR in baseband from EVM

In the logarithmic scale:

$$\frac{S}{N}[dB] \approx 10 \cdot \log_{10} \frac{1}{\left(\frac{EVM[\%]}{100}\right)^2} = -20 \cdot \log_{10} \left(\frac{EVM[\%]}{100}\right)$$

Equation 10-3: Estimation of SNR in baseband from EVM (in dB)

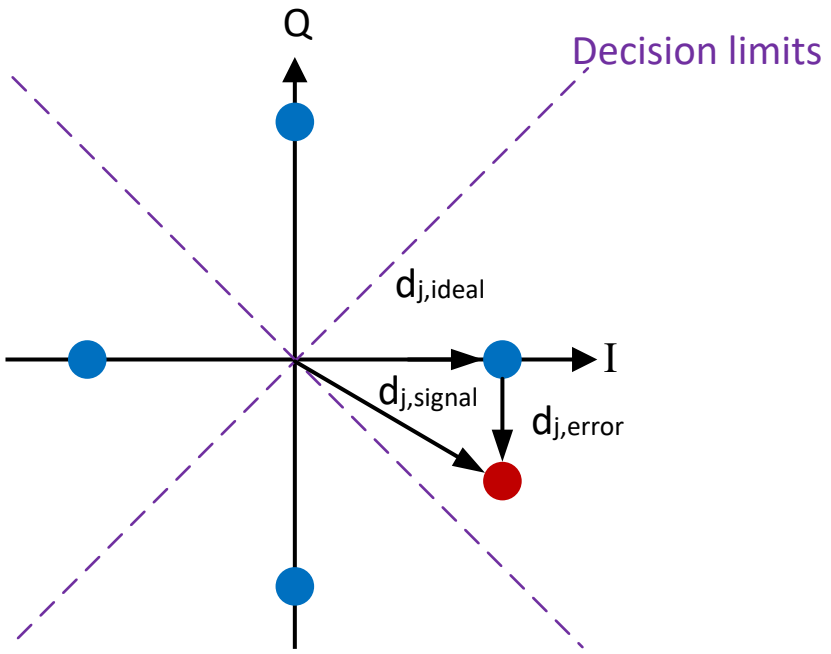

Figure 10-1: Estimation of SNR using error vectors

Figure 10-1 additionally shows the limits of the subsequent decision-maker stage for a QPSK.