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STANDARD-COMPLIANT USAGE OF QUANTITIES, UNITS AND EQUATIONS

FOREWORD

During their studies or vocational training, natural scientists, engineers and technicians usually learn how to correctly use physical quantities, units and equations. In their professional lives, however, this often fades into the background. This brochure, which will refresh this knowledge and serve as a reference, provides an overview of the relevant international standards. It exclusively deals with the units of the International System of Units (abbreviated as SI from the French “Système international d'unités“) and the quantities of the International System of Quantities (ISQ). In addition, it describes the units and quantities that can be used in accordance with the SI or ISQ as well as the logarithmic quantities with decibel as the unit. For other units and quantities, see references [3] and [4].

LEGAL UNITS AND THE INTERNATIONAL SYSTEM OF QUANTITIES

The International System of Units (SI) was adopted in 1960 by the General Conference on Weights and Measures (Conférence Générale des Poids et Mesures, CGPM). The current status is specified in the French version of the SI Brochure published by the International Bureau of Weights and Measures (Bureau International des Poids et Mesures, BIPM) [1].

The SI units have been adopted as the legal units in almost all countries worldwide.

The national standards have been agreed upon with the competent international organizations (ISO and IEC) and describe the internationally recognized state of the art. For the current international standards, see references [3] and [6] to [8]. References [3], [4] and [5] define the SI units on the basis of the International System of Quantities (ISQ).

While the laws only apply to business and official use, the relevant standards are valid without this restriction.

The ISQ base quantities and the SI base units are listed in Table 1, and the derived quantities with special unit symbols in Table 2. Table 3 provides examples of derived quantities without special unit symbols. Table 4 contains the prefixes and prefix symbols for decimal submultiples and multiples of units. Prefixes and prefix symbols are exclusively used together with unit names and unit symbols. Prefix symbols and unit symbols are not separated by a space; together, they form the symbol for a new unit. Table 5 contains examples of the use of prefixes and prefix symbols.

The derived SI units are defined as products, quotients and powers of the base units, the numerical factor always being 1. This system of units is coherent. Table 6 contains units outside the SI that can be used in connection with the SI.

Table 1: ISQ base quantities and SI base units

ISQ base quantity		SI base unit	
Name	**Letter symbol**	**Name**	**Unit symbol**
Length	l	meter	m
Mass	m	kilogram	kg
Time, duration	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T, Q	kelvin	K
Amount of substance	n, v	mole	mol
Luminous intensity	I_v	candela	cd

The letter symbols for the SI units are standardized internationally and are identical in all languages. They must be written as stipulated by law and standard and may not be modified by appending additional information such as indices or attachments.

Table 2: Derived quantities and derived units with special unit symbols

Derived ISQ quantity		Derived SI unit			
Name [1]	Letter symbol	Name	Unit symbol	Expressed in terms of	
				other SI units	SI base units
Plane angle	$\alpha, \beta, \gamma, \varphi$	radian	rad		1
Solid angle	Ω	steradian	sr		1
Frequency	f, ν	hertz	Hz		s^{-1}
Force	F	newton	N		$m\ kg\ s^{-2}$
Pressure, stress	P	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
Energy, work, amount of heat	W	joule	J	N · m	$m^2\ kg\ s^{-2}$
Power, radiant flux	P	watt	W	J/s	$m^2\ kg\ s^{-3}$
Voltage, electric tension	U, V	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
Electric charge	Q	coulomb	C		A s
Electric capacitance	C	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
Electric resistance	R	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
Electric conductance	G	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
Magnetic flux	Φ	weber	Wb	V · s	$m^2\ kg\ s^{-2}\ A^{-1}$
Magnetic flux density	B	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
Inductance	L	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Luminous flux	Φ_v	lumen	lm	cd · sr	cd
Illuminance	E_v	lux	lx	lm/m^2	m^{-2} cd
Activity referred to a radionuclide	A	becquerel	Bq		s^{-1}
Absorbed dose	D	gray	Gy	J/kg	$m^2\ s^{-2}$
Dose equivalent	H	sievert	Sv	J/kg	$m^2\ s^{-2}$
Catalytic activity		katal	kat		$mol\ s^{-1}$

Table 3: Examples of derived quantities without special unit symbols

Derived ISQ quantity		Derived SI unit		
Name	Letter symbol	Name	Expressed in terms of	
			other SI units	SI base units
Area	A	square meter		m^2
Volume	V	cubic meter		m^3
Speed, velocity	v	meter per second		$m\ s^{-1}$
Acceleration	a	meter per second squared		$m\ s^{-2}$
Angular velocity	ω	radian per second	rad/s	s^{-1}
Angular acceleration	α	radian per second squared	rad/s^2	s^{-2}
Moment of force	M	newton meter	N · m	$m^2\ kg\ s^{-2}$
Heat flux density	q	watt per square meter	W/m^2	$kg\ s^{-3}$
Heat capacity	C	joule per kelvin	J/K	$m^2\ kg\ s^{-2}\ K^{-1}$
Thermal conductivity	λ	watt per meter kelvin	W/(m · K)	$m\ kg\ s^{-3}\ K^{-1}$
Energy density	e	joule per cubic meter	J/m^3	$m^{-1}\ kg\ s^{-2}$
Electric field strength	E	volt per meter	V/m	$m\ kg\ s^{-3}\ A^{-1}$
Magnetic field strength	H	ampere per meter		$A\ m^{-1}$

[1] If it is absolutely clear from the context that quantities of electricity are being referred to, the adjective "electric" can be omitted.

Table 4: Prefixes and prefix symbols for decimal submultiples and multiples of units

Prefix	Symbol	Factor
yocto	y	10^{-24}
zepto	z	10^{-21}
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	µ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deca	da	10^{1}
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}
zetta	Z	10^{21}
yotta	Y	10^{24}

When referring to mass, the prefixes must be added to the gram.

Table 5: Examples of the use of prefixes and prefix symbols

Unit	Unit name	Relation
km	kilometer	1 km = 10^{3} m
mm	millimeter	1 mm = 10^{-3} m
µm	micrometer	1 µm = 10^{-6} m
nm	nanometer	1 nm = 10^{-9} m
TW	terawatt	1 TW = 10^{12} W
GW	gigawatt	1 GW = 10^{9} W
MW	megawatt	1 MW = 10^{6} W
kW	kilowatt	1 kW = 10^{3} W
mW	milliwatt	1 mW = 10^{-3} W
µW	microwatt	1 µW = 10^{-6} W
nW	nanowatt	1 nW = 10^{-9} W
pW	picowatt	1 pW = 10^{-12} W

Table 6: Non-SI units accepted for use with the SI

Quantity	Unit name	Unit symbol	Relation	Value in SI units
Time	minute	min		1 min = 60 s
	hour	h	1 h = 60 min	1 h = 3600 s
	day	d	1 d = 24 h	1 d = 86 400 s
Plane angle	degree	°		1° = (π/180) rad
	minute of an arc	´	1´ = (1/60)°	1´ = (π/10 800) rad
	second of an arc	″	1″ = (1/60)´	1″ = (π/648 000) rad
Area	hectare	ha		1 ha = 10^{4} m^{2}
Volume	liter	L, l		1 L = 10^{-3} m^{3}
Mass	ton	t		1 t = 10^{3} kg
Pressure	bar	bar	10^{5} Pa	1 bar = 10^{5} m^{-1} kg s^{-2}

QUANTITIES

Physical phenomena are described qualitatively and quantitatively by physical quantities. Every value of a quantity can be expressed as the product of numerical value and unit. If the unit changes (for example, by adding a prefix symbol), the numerical value changes as well. The product of numerical value and unit remains constant; it is invariant with respect to a change of unit.

Example: U = 0.1 V and U = 100 mV describe the same quantity value.

Letter symbols for physical quantities are specified in the international IEC 60027 standards ([6], [7] and [8]) and in IEV 112 [5].

Multiple-letter abbreviations of names should not be used as quantity symbols. When it is necessary to indicate a special meaning of a quantity symbol, letters or numerals can be added to the general quantity symbol as indices.

Letter symbols for quantities should not contain any reference to specific units.

Quantities of the same kind are specified in the same unit and are distinguished either by different letter symbols or by letter symbols with index. Tables 7, 8 and 9 contain some examples of quantities of the same kind. Only quantities of the same kind can be added to or subtracted from each other.

Quantities can be multiplied or divided in order to define additional quantities (see Table 10 for examples).

Quantities can be multiplied or divided using numerical factors.

Table 7: Examples of quantities of the kind of quantity referred to as length

Quantity		Unit	
Name	**Letter symbol**	**Name**	**Letter symbol**
Length	l	meter	m
Width	b	meter	m
Height	h	meter	m
Thickness	D, d	meter	m
Radius	r, R	meter	m
Diameter	d, D	meter	m
Circumference	u, U	meter	m
Wavelength	λ	meter	m

Table 8: Examples of quantities of the kind of quantity referred to as power

Quantity		Unit	
Name	**Letter symbol**	**Name**	**Letter symbol**
Power	P	watt	W
Signal power	P_s	watt	W
Noise power	P_n	watt	W
Active power	P, P_p	watt	W
Reactive power	Q, P_q	watt	W (also var)
Apparent power	S, P_a	watt	W (also VA)

Table 9: Examples of quantities of the kind of quantity referred to as voltage or electric tension

Quantity		Unit	
Name	**Letter symbol**	**Name**	**Letter symbol**
Voltage, electric tension	U, u	volt	V
RMS value of an alternating voltage	U_RMS	volt	V
Peak value of an alternating voltage	U_p	volt	V
Rectified value of an alternating voltage	U_m	volt	V
Complex amplitude of a sine voltage	$\underline{U}$	volt	V

Table 10: Examples of derived ISQ quantities

Name [1]	Letter symbol	Expressed in terms of	
		other ISQ quantities	ISQ base quantities
Area	A		l^2
Volume	V		l^3
Speed, velocity	v		$l\ t^{-1}$
Acceleration	a		$l\ t^{-2}$
Force	F		$l\ m\ t^{-2}$
Pressure, stress	p	F/l^2	$l^{-1}\ m\ t^{-2}$
Energy, work, amount of heat	W	$F \cdot l$	$l^2\ m\ t^{-2}$
Power, radiant flux	P	W/t	$l^2\ m\ t^{-3}$
Voltage, electric tension	U, V	P/I	$l^2\ m\ t^{-3}\ I^{-1}$
Electric charge	Q		$I\ t$
Electric capacitance	C	Q/U	$l^{-2}\ m^{-1}\ t^4\ I^2$
Electric resistance	R	U/I	$l^2\ m\ t^{-3}\ I^{-2}$
Magnetic flux	Φ	$U \cdot t$	$l^2\ m\ t^{-2}\ I^{-1}$
Magnetic flux density	B	$\Phi\ /l^2$	$m\ t^{-2}\ I^{-1}$
Inductance	L	Φ/I	$l^2\ m\ t^{-2}\ I^{-2}$
Moment of force	M	$F \cdot l$	$l^2\ m\ t^{-2}$
Heat flux density	q	P/l^2	$m\ t^{-3}$
Heat capacity	C	W/T	$l^2\ m\ t^{-2}\ T^{-1}$
Thermal conductivity	λ	$P/(l \cdot T)$	$l\ m\ t^{-3}\ T^{-1}$
Energy density	e	W/l^3	$l^{-1}\ m\ t^{-2}$
Electric field strength	E	U/l	$l\ m\ t^{-3}\ I^{-1}$
Magnetic field strength	H	I/l	$I\ l^{-1}$

[1] If it is absolutely clear from the context that quantities of electricity are being referred to, the adjective “electric” can be omitted.

QUANTITIES WITH REFERENCE VALUES, LEVEL QUANTITIES

Level quantities indicate the difference to a defined reference value that is other than zero. Every value of a level quantity can be expressed as the product of numerical value and unit. The letter symbol for the level quantity must additionally include a note relating to the reference quantity, or a special letter symbol must be used.

For two values each of a level quantity, it is possible to form a difference that is a quantity. The difference quantity is independent of the reference value.

The temperature difference can be indicated in kelvin or degree Celsius; the values are identical.

Adding the values of a level quantity only makes sense if the sum is divided by the number of addends. This yields the mean value, which is also a level quantity.

A quantity of the same kind can be added to or subtracted from a level quantity. The result is also a level quantity.

Table 11: Examples of level quantities

Level quantity	Letter symbol	Reference value	Unit	Unit symbol
Height coordinate of a terrain point	h_{NHN}	national benchmark, standard zero level	meter	m
Water level coordinate of a water body	h_{P}	staff gage zero	meter	m
Electric potential	φ	zero potential, e.g. ground potential	volt	V
Time of day	t_{d}	midnight, 0:00 h	second, minute, hour	s, min, h
Celsius temperature	T_{C}	T_0 = 273.15 K	degree Celsius	°C

Table 12: Examples of level quantity differences

Level quantity	Difference	Difference quantity	Unit	Unit symbol
Height coordinate of a terrain point	$\Delta h = h_{\mathrm{NHN},2} - h_{\mathrm{NHN},1}$	height difference	meter	m
Water level coordinate of a water body	$\Delta h = h_{\mathrm{P},2} - h_{\mathrm{P},1}$	level difference	meter	m
Electric potential	$U = \Delta\varphi = \varphi_2 - \varphi_1$	voltage, electric tension	volt	V
Time of day	$\Delta t_{\mathrm{d}} = t_{\mathrm{d},2} - t_{\mathrm{d},1}$	duration	second, minute, hour	s, min, h
Celsius temperature	$\Delta T_{\mathrm{C}} = T_{\mathrm{C},2} - T_{\mathrm{C},1}$	temperature difference	degree Celsius, kelvin	°C, K

Table 13: Examples of mean values of level quantities

Level quantity	Mean value	Mean level	Unit	Unit symbol
Height coordinate of a terrain point	$h_{\mathrm{m}} = (h_{\mathrm{NHN},2} + h_{\mathrm{NHN},1})/2$	mean height coordinate	meter	m
Water level coordinate of a water body	$h_{\mathrm{m}} = (h_{\mathrm{P},2} + h_{\mathrm{P},1})/2$	mean water level coordinate	meter	m
Electric potential	$\varphi_{\mathrm{m}} = (\varphi_2 + \varphi_1)/2$	mean potential	volt	V
Time of day	$t_{\mathrm{d,m}} = (t_{\mathrm{d},2} + t_{\mathrm{d},1})/2$	mean time	second, minute, hour	s, min, h
Celsius temperature	$T_{\mathrm{C,m}} = (T_{\mathrm{C},2} + T_{\mathrm{C},1})/2$	mean temperature	degree Celsius	°C

PHYSICAL CONSTANTS

Every value of a physical constant can be expressed as the product of numerical value and unit. In equations, it can be treated like a quantity.

Table 14: Examples of fundamental physical constants

Name	Letter symbol	Relation	Expressed in SI units	
Speed of light in vacuum	c, c_0			$299792458 \dfrac{\text{m}}{\text{s}}$
Magnetic field constant	μ_0		$\dfrac{4\pi}{10} \dfrac{\mu\text{H}}{\text{m}}$	$4\pi \cdot 10^{-7} \dfrac{\text{V s}}{\text{A m}}$
Electric field constant	ε_0	$\varepsilon_0 = \dfrac{1}{\mu_0 c^2}$	$8.854 \dfrac{\text{pF}}{\text{m}}$	$8.854 \cdot 10^{-12} \dfrac{\text{A s}}{\text{V m}}$
Field characteristic impedance	Z_0	$Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \mu_0 c$		$376.730\ \Omega$
Elementary charge	e		$1.602 \cdot 10^{-19}\,\text{C}$	$1.602 \cdot 10^{-19}\,\text{A s}$
Electron mass	m_e			$9.109 \cdot 10^{-31}$ kg
Gravitational constant	G			$6.674 \cdot 10^{-11} \dfrac{\text{m}^3}{\text{kg s}^2}$
Boltzmann constant	k_B			$1.381 \cdot 10^{-23} \dfrac{\text{J}}{\text{K}}$
Loschmidt constant	N_L			$2.687 \cdot 10^{25}\ \text{m}^{-3}$

Note: The value for the speed of light is precise, whereas all other numerical values are rounded.
The values of numerous fundamental physical constants can be found in [11] and [12].

EQUATIONS

The terms quantity equation, scaled quantity equation and numerical value equation as well as the relation "value of quantity equals numerical value times unit" are based on the work of Julius Wallot and others between 1922 and 1933. In 1931, discussions about this topic led to the first edition of the German DIN 1313 standard on the notation of physical equations. For the current edition of this standard, see [9].

In equations, the letter symbols for quantities, constants and units must be treated like algebraic variables.

Quantity equations

Quantity equations are equations where the letter symbols represent physical quantities or mathematical symbols ([2], [3], [5]). These equations are independent of the selected units. When evaluating quantity equations, the products of numerical value and unit must be substituted for the letter symbols. Numerical values and units in quantity equations are treated as independent factors.

Example: The equation

$$U = R \cdot I$$

always yields the same result for voltage U, irrespective of the units in which resistance R and current I are expressed, provided that the associated products of numerical value and unit are substituted for R and I.

Scaled quantity equations

Scaled quantity equations are quantity equations where every quantity appears with its unit in the denominator [9].

Examples:

$$U/\mathrm{V} = (R/\Omega) \cdot (I/\mathrm{A})$$

$$U/\mathrm{V} = (R/\mathrm{k\Omega}) \cdot (I/\mathrm{mA})$$

$$U/\mathrm{kV} = 10^{-3} \cdot (R/\Omega) \cdot (I/\mathrm{A})$$

The parentheses can be omitted if the assignment of quantities and units is clear without parentheses, for example on the left side of the above equations or when horizontal fraction lines are used:

$$\frac{U}{\mathrm{kV}} = 10^{-3} \cdot \frac{R}{\Omega} \cdot \frac{I}{\mathrm{A}}$$

The advantage of a scaled quantity equation is that the quotients of quantity and unit directly represent the numerical values. However, the equations remain correct even if the products of numerical value and unit in other units are substituted for the quantities. In this case, however, the units must be converted. The use of the scaled quantity equation is recommended for representing results.

Numerical value equations

Numerical value equations are equations where the letter symbols represent the numerical values of physical quantities or mathematical symbols. These equations are dependent on the selected units.

Numerical value equations should no longer be used because they are considered outdated. According to ISO 80000-1 [3] they must be indicated as numerical value equations; units must be specified for all quantities.

When coherent units are used, the numerical value equations coincide with the corresponding quantity equations. When identical letter symbols are used for quantities and numerical values, it is not possible to distinguish between a quantity equation and a numerical value equation.

To indicate the numerical value and the unit of a quantity, the standards use braces or brackets with the following meaning:

$\{U\}$	numerical value of quantity U
$[U]$	unit of quantity U
$U = \{U\} \cdot [U]$	quantity = numerical value · unit

This notation is only required when units outside the SI are used. According to the relevant standards, it is not allowed to add units in brackets to quantity symbols in equations or to write the units in brackets in front of, beside or under the equations.

Examples of incorrect usage:

U [kV] = $10^{-3} \cdot R$ [Ω] $\cdot\ I$ [A] incorrect

$U = 10^{-3} \cdot R \cdot I$ U [kV], R [Ω], I [A] incorrect

Such notations must never be used. To show the relation between numerical values, the scaled quantity equation is the preferred form. The following notations are correct: U in kV, R in Ω, I in A. These notations have the advantage that they are identical in English and German.

Example:

$U = 10^{-3} \cdot R \cdot I$ U in kV, R in Ω, I in A
(correct but not recommended)

Equations with level quantities

For level quantities with special units, scaled quantity equations or numerical value equations should be used.

Example: conversion of a thermodynamic temperature into the corresponding Celsius temperature:

As a scaled quantity equation:

T_C/°C = T/K – 273.15

As a numerical value equation:

$T_C = T - 273.15$ T_C in °C , T in K

LOGARITHMIC RATIOS OF QUANTITIES, ATTENUATION AND GAIN FIGURES

In telecommunications and acoustics, an attenuation or gain figure describes the logarithmic ratio of two electrical quantities of the same kind that identifies the characteristics of a two-port or of a transmission path. The unit used is the decibel (dB). The arguments of the logarithm are numerical values. The unit dB is not an SI unit but – like SI units – should not be modified by appending additional information. The function "lg" describes the logarithm to base 10; "log" is the general logarithm function.

In the following equations, the index 1 designates the input quantity, whereas the index 2 designates the output quantity of a two-port.

Definition for power quantities

Example: active power

Power attenuation figure of a two-port:

$$A_P = \left(10 \text{ lg } \frac{P_1}{P_2}\right) \text{dB}$$

Power gain figure of a two-port:

$$G_P = \left(10 \text{ lg } \frac{P_2}{P_1}\right) \text{dB} = -A_P$$

Definition for quantities whose square is proportional to a power quantity

Example: complex amplitudes or RMS values of alternating voltages

Voltage attenuation figure of a two-port:

$$A_U = \left(20 \text{ lg } \left|\frac{\underline{U}_1}{\underline{U}_2}\right|\right) \text{dB} = \left(20 \text{ lg } \frac{U_{1\,\text{RMS}}}{U_{2\,\text{RMS}}}\right) \text{dB}$$

Voltage gain figure of a two-port:

$$G_U = \left(20 \text{ lg } \left|\frac{\underline{U}_2}{\underline{U}_1}\right|\right) \text{dB} = \left(20 \text{ lg } \frac{U_{2\,\text{RMS}}}{U_{1\,\text{RMS}}}\right) \text{dB}$$

These quantities used to be called field quantities, but this designation was misleading. Power and energy densities are both field and power quantities. Electric voltage and electric current are not field quantities but integrals over field quantities. Therefore, ISO 80000-1 [3] introduced the designation "root power quantity". This designation was adopted in the draft for the next version of the IEC 60027 standard.

LOGARITHMIC RATIOS OF QUANTITIES, LEVEL

In telecommunications and acoustics, the logarithmic ratio of two quantities is defined as a level when the denominator is a fixed value of a reference quantity of the same kind as the numerator [8]. The unit used is the decibel (dB). The value of the reference quantity should always be specified for numerical values of levels. To abbreviate this specification, the reference quantity in parentheses can follow the dB symbol. If the numerical value of the reference quantity equals 1, it can be omitted in the parentheses. To make clear that this is not a special unit but only a designation for the reference value, a space should separate the dB symbol and the expression in parentheses (see [8]). Reference [8] also mentions some abbreviations introduced by the International Telecommunication Union (ITU) [10]. In these abbreviations, dB is directly followed by a letter or a sequence of characters to identify the reference value. IEC 60027-3 ([8]) recommends not to use these abbreviations.

Definition for power quantities

Example: power P, reference value P_0

$$L_P(\text{re } P_0) = L_{P/P_0} = 10 \text{ lg } \frac{P}{P_0} \text{ dB}$$

Definition for quantities whose square is proportional to a power quantity

Example: voltage U, reference value U_0

$$L_U(\text{re } U_0) = L_{U/U_0} = 20 \text{ lg } \frac{U}{U_0} \text{ dB}$$

Table 15 contains some level definitions and their short forms. Other level definitions that are common in telecommunications are listed in IEC 60027-2 [7].

The short forms in columns 5 and 6 of Table 15 are only suitable for indicating measured values and results. In general, signs denoting reference values and measurement methods should be appended to the quantity symbol and not to the unit symbol. This applies not only to the SI units but also to the decibel. In acoustics, the previously common units including additional information (e.g. dB(A)) are no longer used.

Table 15: Examples of level definitions with different reference quantities

Quantity, reference value	Letter symbol		Level, definition	Unit, short form	
	Long form	Short form		IEC [1]	ITU [2]
Electric power, reference value: 1 mW	L_P (re 1 mW)	$L_{P/\text{mW}}$	$10 \text{ lg} \left(\frac{P}{1 \text{ mW}}\right) \text{ dB}$	dB (mW)	dBm
Voltage, electric tension, reference value: 1 V	L_U (re 1 V)	$L_{U/\text{V}}$	$20 \text{ lg} \left(\frac{U_{\text{RMS}}}{1 \text{ V}}\right) \text{ dB}$	dB (V)	dBV
Voltage, electric tension, reference value: 1 µV	L_U (re 1 µV)	$L_{U/\mu\text{V}}$	$20 \text{ lg} \left(\frac{U_{\text{RMS}}}{1\ \mu\text{V}}\right) \text{ dB}$	dB (µV)	dBµV
Electric current, reference value: 1 µA	L_I (re 1 µA)	$L_{I/\mu\text{A}}$	$20 \text{ lg} \left(\frac{I_{\text{RMS}}}{1\ \mu\text{A}}\right) \text{ dB}$	dB (µA)	dBµA
Electric field strength, reference value: 1 µV/m	L_E (re 1 µV/m)	$L_{E/(\mu\text{V/m})}$	$20 \text{ lg} \left(\frac{E_{\text{RMS}}}{1\ \mu\text{V/m}}\right) \text{ dB}$	dB (µV/m)	not: dBµV/m [3]
Magnetic field strength, reference value: 1 µA/m	L_H (re 1 µA/m)	$L_{H/(\mu\text{A/m})}$	$20 \text{ lg} \left(\frac{H_{\text{RMS}}}{1\ \mu\text{A/m}}\right) \text{ dB}$	dB (µA/m)	not: dBµA/m [3]
Relative noise level Carrier power: P_c Spurious signal power: P_n	L_n (re P_c)	L_{n,P_c}	$10 \text{ lg} \left(\frac{P_n}{P_c}\right) \text{ dB}$	dB (P_c)	dBc

[1] To be replaced by dB (without additional information) in quantity equations.
[2] These short forms should be avoided.
[3] These short forms are incorrect.

Relation between electric and magnetic field strength levels

The field strengths are linked by the equation $E_{\mathrm{RMS}} = Z_{\mathrm{C}} \cdot H_{\mathrm{RMS}}$, where Z_{C} is the characteristic impedance.

Conversion into level:

$$20 \lg \left(\frac{E_{\mathrm{RMS}}}{1\,\mu\mathrm{V/m}} \right) \mathrm{dB} = 20 \lg \left(\frac{H_{\mathrm{RMS}}}{1\,\mu\mathrm{A/m}} \cdot \frac{Z_{\mathrm{C}}}{1\,\Omega} \right) \mathrm{dB}$$

$$= 20 \lg \left(\frac{H_{\mathrm{RMS}}}{1\,\mu\mathrm{A/m}} \right) \mathrm{dB} + 20 \lg \left(\frac{Z_{\mathrm{C}}}{1\,\Omega} \right) \mathrm{dB}$$

The expression

$$A_{Z/\Omega} = 20 \lg \left(\frac{Z_{\mathrm{C}}}{1\,\Omega} \right) \mathrm{dB}$$

and the letter symbols in Table 15 can be used to describe the relation between the field strength levels as follows:

$$L_{E/(\mu\mathrm{V/m})} = L_{H/(\mu\mathrm{A/m})} + A_{Z/\Omega}$$

where $A_{Z/\Omega}$ is the impedance conversion figure (suggested designation). This is not a level because the reference value is neither a power quantity nor a root power quantity. dB (Ω) can be used as the short form; dBΩ should be avoided. The characteristic impedance Z_{C} can be a line characteristic impedance or the field characteristic impedance Z_0 (Table 14).

MATHEMATICAL OPERATIONS FOR LOGARITHMIC RATIOS OF QUANTITIES

In equations, a logarithmic attenuation or gain figure should be treated like a quantity and a logarithmic level like a level quantity.

The mean value of two levels with identical reference values is also a level:

$$L_\mathrm{m} = (L_1 + L_2)/2$$

The difference between two levels with identical reference value is an attenuation or gain figure:

$$\Delta L = A = L_1 - L_2$$

Attenuation or gain figures can be added to and subtracted from each other or multiplied by and divided by real factors:

$$A_\mathrm{s} = A_1 + A_2, \quad A_\mathrm{d} = A_1 - A_2, \quad A_\mathrm{p} = \mathrm{k} \cdot A_1, \quad A_\mathrm{q} = A_1/\mathrm{k}$$

Attenuation or gain figures can be added to or subtracted from levels:

$$L_1 = L_2 + A, \quad L_1 = L_2 - A$$

This mathematical operation only complies with the rules of algebra if both the level and the attenuation or gain figure are specified in dB without any additional information.

These relations should be regarded as quantity equations. The values for level and attenuation or gain figures must be entered as products of numerical value and the unit dB without any additional information.

If ITU-compliant units are used for the level (e.g. dBm), the equation for the level difference can either be written as a scaled quantity equation

$$\Delta L/\mathrm{dB} = A/\mathrm{dB} = L_1/\mathrm{dBm} - L_2/\mathrm{dBm}$$

or as a numerical value equation:

$\Delta L = A = L_1 - L_2$ ΔL, A in dB, L_1, L_2 in dBm
(correct but not recommended)

The mean value is obtained from the scaled quantity equation:

$$L_\mathrm{m}/\mathrm{dBm} = (L_1/\mathrm{dBm} + L_2/\mathrm{dBm})/2$$

NOTATION

The notation for quantities and units is standardized internationally in [3]; see also [1] and [2].

Italics

The following are written in italic (sloping) type:

- Letter symbols for physical quantities, e.g. m (mass); U (electric voltage)
- Letter symbols for variables, e.g. x; n
- Symbols for functions and operators with user-definable meaning, e.g. $f(x)$

The standards recommend that a serif font (e.g. Times) be used for these letter symbols.

Roman type

The following are written in roman (upright) type:

- Units and their prefixes, e.g. m; mm; kg; s; MW; µV; dB
- Letter symbols for constants, e.g. c (speed of light)
- Numerals, e.g. 4.5; 67; 8-fold; 1/2
- Symbols for functions and operators with fixed meaning, e.g. sin; lg; π
- Chemical elements and compounds, e.g. Cu; H_2O

The standards do not recommend a special font for these letter symbols.

For numerals, a font such as Arial should be used to clearly distinguish the numerals "one" (1) and "zero" (0) from a lowercase L ("l") or an uppercase i ("I") and an uppercase o ("O").

Letter symbols for units are written in lowercase (e.g. m, s), unless they are derived from a name (e.g. A, W). If unit prefixes indicate decimal submultiples, they are written in lowercase. If unit prefixes indicate decimal multiples, they are written in uppercase – except for k.

QUANTITY VALUES IN TABLES AND DIAGRAMS

Tables 16 and 17 show examples of standard-compliant and incorrect labeling of table headers and coordinate systems.

The labeling used for the displays of electronic equipment, in particular of test and measurement instruments, should also follow these recommendations. The extensive functionality of state-of-the-art electronic equipment often causes problems because space and character set are limited. Therefore, it is sometimes necessary to make compromises.

Table 16: Labeling of table headers and coordinate systems

Correct				Incorrect [1]
U	U/V, U in V	E	E/(µV/m), E in µV/m	U [V], U in [V]
1 V	1	1 µV/m	1	1
2 V	2	2 µV/m	2	2
3 V	3	3 µV/m	3	3
…	…	…	…	…

[1] Do not put units in brackets.

Table 17: Labeling of table headers and coordinate systems for large value ranges

Correct			Incorrect [1]
P	P/W	P/W	P/W
1 TW	$1 \cdot 10^{12}$	10^{12}	1 T
1 GW	$1 \cdot 10^{9}$	10^{9}	1 G
1 MW	$1 \cdot 10^{6}$	10^{6}	1 M
1 kW	$1 \cdot 10^{3}$	10^{3}	1 k
1 W	1	1	1
1 mW	$1 \cdot 10^{-3}$	10^{-3}	1 m
1 µW	$1 \cdot 10^{-6}$	10^{-6}	1 µ
1 nW	$1 \cdot 10^{-9}$	10^{-9}	1 n
1 pW	$1 \cdot 10^{-12}$	10^{-12}	1 p

[1] Do not use prefixes alone.

FREQUENT MISTAKES

Many articles in technical journals, documentation and papers do not comply with the correct usage of quantities, units and equations as specified by the relevant national and international standards.

It is common bad practice to add indices to units. This practice violates the relevant standards. An index must always be appended to the quantity symbol, not to the unit symbol. As a result of this incorrect usage, units are converted when conversions of quantities are referred to. In this context, the decibel (dB) causes particular problems. All these problems can be avoided if quantities are defined instead of special units and the reference value is appended to the quantity symbol as an index.

Another case of noncompliance with standards is placing the unit in brackets next to the quantity symbol. This bad practice is unfortunately very common. A scaled quantity equation should be used when both the unit and the quantity are to be indicated.

Dr. Klaus H. Blankenburg, July 2017

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[5]	IEC 60050-112:2010, IEV PART 112: QUANTITIES AND UNITS (E/F) (www.electropedia.org)
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[8]	IEC 60027-3:2004, Letter symbols to be used in electrical technology – Part 3: Logarithmic and related quantities, and their units
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[11]	Fundamental Physical Constants – Extensive Listing (http://physics.nist.gov/constant)
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Designations of the cited institutions

CGPM Conférence générale des poids et mesures (General Conference on Weights and Measures)

JCGM Joint Committee for Guides in Metrology

BIPM Bureau International des Poids et Mesures (International Bureau of Weights and Measures)

ISO International Organization for Standardization

IEC International Electrotechnical Commission

CENELEC Comité Européen de Normalisation Electrotechnique (European Committee for Electrotechnical Standardization)

DIN Deutsches Institut für Normung (German Institute for Standardization)

ITU International Telecommunication Union

NIST National Institute of Standards and Technology

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