Probability of intercept for frequency hop signals using search receivers (II)

3.3 Receiver in wait mode

Given that the hopper’s frequency spacing is known from searching the frequency range, a receiver can be tuned to one of the hop channels. The FH transmitter is then detected when its instantaneous frequency coincides with the set receive frequency. For a hop to be detectable its duration must be

\[ T_h > T_i. \]

In wait mode (hopper trap) the time required for synthesizer setting and signal processing does not influence probability of intercept, so \( T_d = T_i \) applies in this case.

Provided \( T_i < T_h \) is valid and the condition described under 2 is met, exactly one detection occurs when the hop frequency and receive frequency coincide. On arrival of a burst the level threshold is exceeded as soon as the detection filter has settled, and at the end of the burst after a corresponding delay the level drops below the threshold again, allowing estimation of the burst duration. In this mode probability of intercept does not depend on hop duration \( T_h \) and the arrival time of the burst at the receiving antenna, as is the case with a search receiver in hop mode, so \( n = 1 \) applies. FIG 5b with \( M_d = M_{Sc} = 1 \) and the equations (5) and (4) are valid for the probability of intercept of a single-channel and a multichannel receiver.

If the mean number of valid detection attempts per burst of a sufficiently fast search receiver is \( \bar{n} > 1 \), the probability of intercept is greater than in wait mode (equation 8). The probability of intercept in search mode may also be less than in wait mode however:

a) When the receiver scan is too slow compared with burst duration \( (T_i < T_h < (T_d + T_i)) \), in which case the factor \( \bar{n} = (T_h - T_i) / T_d \) in (8) is less than 1.

b) If the search receiver is unable to use the a priori information on the hopper frequency range that was assumed for the receiver in wait mode, the receiver may also search in frequency ranges not used by the hopper (FIGs 5a and c). In this case the ratio \( M_d / M_{Sc} \) in (8) is less than 1 while in wait mode \( M_d = M_{Sc} = 1 \) holds.

4 Interception of frequency hoppers: repeated attempts

Up to now we dealt with the probability of intercepting a single hop (burst). If an FH transmitter can be observed for a certain operating time \( T_h \) (transmit time of hopper or total on time of receiver), the attempt to hit it can be repeated at \( N \) hops with

\[ N = T_h f_H \]  

if \( f_H \) is the hop rate of the transmitter in frequency hops per time increment. (Note: \( f_H \) does not equal \( 1 / T_h \) as synthesizer settling has to be considered for the transmitter too.) With each of the \( N \) attempts the probability of a hit is \( P \), with \( P = P_1 \) in (3 to 6) or \( P_{1h} \) in (8).

4.1 Binomial distribution

The probability \( P_N \) that in \( N \) attempts a number \( Z \) of exactly \( k \) hits occurs is calculated according to the binomial distribution [5]:

\[ P_N (Z = k) = \binom{N}{k} P^k (1 - P)^{N-k} \]

with

\[ \binom{N}{k} = \frac{N!}{k!(N-k)!} \]  

and the mean value (mean number of hits)

\[ \bar{k} = N \times P \]  

Of particular interest are the probabilities derived from (15) that the number \( Z \) of hits occurs within a defined interval:

a) The probability of at least one hit in \( N \) attempts is

\[ P_N (Z \geq 1) = 1 - (1 - P)^N \]  

b) The probability of \( k_1 \) to \( k_2 \) hits in \( N \) attempts is

\[ P_N (k_1 \leq Z \leq k_2) = \sum_{l=k_1}^{k_2} \binom{N}{l} P^l (1 - P)^{N-l} \]  

c) The probability of at least \( k \) hits in \( N \) attempts is

\[ P_N (Z \geq k) = 1 - \sum_{l=0}^{k-1} \binom{N}{l} P^l (1 - P)^{N-l} \]

4.2 Poisson theorem

With a large number of hopper channels \( M_{HH} \), the probability \( P \) of intercept for a single hop is often very low, so that even with a large number \( N \) of attempts the product \( N \times P \) is not a very high number but of the order of 1. In this case the binomial distribution (15) for \( k \) of the order of \( N \times P \) may be approximated by the Poisson distribution [5]:

\[ P_N (Z = k) \approx \frac{e^{-NP} (NP)^k}{k!} \]  

A special case should be mentioned:

Given a large number of hopper channels \( M_{HH} \) and the search range of a single-channel receiver coinciding with the hop range of the transmitter
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\[ M_{SC} = M_{FH}, \text{FIG 5d}, \] L complete scans are to be performed [4]. The receiver is to be able to perform on average just one valid detection attempt per frequency hop of the transmitter \((\bar{n} = 1)\). With (5) the detection probability for a single burst is \( P = P_1 = 1/M_{FH} \) and the total number of attempts \( N = L \times M_{FH} \). With \( N \times P = L \times M_{FH} \times (1/M_{FH}) = L \) and using (19) and (20), the probability of at least \( k \) hits (FIG 7) is

\[
P_N (Z \geq k) = 1 - \sum_{i=0}^{k-1} P_N (Z = i) \tag{21}
\]

According to (21) at least one hit occurs (trace \( k = 1 \) in FIG 7) with the probability

\[
P_N (Z \geq 1) = 1 - e^{-1} \tag{22}
\]

43 DeMoivre-Laplace theorem

If the number \( N \) of attempts is sufficiently large that

\[
NP(1 - P) \gg 1 \tag{23}
\]

is obtained, the binomial distribution (15) can be approximated by a Gaussian distribution [5]:

\[
P_N (Z = k) \approx \frac{e^{-(k - NP)^2/2NP(1 - P)}}{\sqrt{2\pi NP(1 - P)}} \tag{24}
\]

For the total probability (18) the following is obtained:

\[
P_N (k_1 \leq Z \leq k_2) \approx \frac{1}{2} \left[ \text{erf} \left( \frac{k_2 - NP}{\sqrt{2NP(1 - P)}} \right) - \text{erf} \left( \frac{k_1 - NP}{\sqrt{2NP(1 - P)}} \right) \right] \tag{25}
\]

where \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy \tag{26} \)

The relationship between the number of attempts and receiver scans is shown in the blue BOX.

5 Example

Let us assume that the search range of the receiver and the frequency range of the FH transmitter coincide (FIG 5d) and that the hopper and receiver have 2000 hop positions each \((M_{FH} = M_{SC} = 2000)\). In this case the probability of intercepting a single burst in a single measurement with a single-channel receiver is \( P_1 = 1/M_{FH} = 1/2000 \) according to (5).

a) Poisson theorem:

With \( L = 3 \) the probability of the hopper being intercepted at least once in three scans is 95\% according to curve \( k = 1 \)

Based on (25) with (19), the probability of at least \( k \) hits in \( N \) attempts is

\[
P_N (Z \geq k) \approx \frac{1}{2} \left[ \text{erf} \left( \frac{N - NP}{\sqrt{2NP(1 - P)}} \right) - \text{erf} \left( \frac{k - NP}{\sqrt{2NP(1 - P)}} \right) \right] \tag{27}
\]

The number of repeated attempts during a number of receiver scans

The number \( N \) of repeated attempts to detect a transmitter with a random distribution of frequency hops is a decisive parameter for determining probability of intercept (15) to (27). The relationship between \( N \) and the number \( L \) of receiver scans is as follows:

The dwell time of the receiver at a frequency is assumed to be \( T_d \). For a systematic search through all frequency positions \( M_{SC} \) of a single-channel receiver, the time \( T_{Sc,1} = M_{SC} T_d \) is required for one scan.

With (14) the number \( N \) of attempts during \( L \) scans is defined as

\[
N_{L,1} = M_{SC} T_d \; \bar{n} \; L \tag{28}
\]

In the case of a multichannel receiver with \( K \) parallel channels the scan time reduces to

\[
T_{Sc,K} = M_{SC} T_d \; \frac{1}{K} \tag{29}
\]

with the result that during \( L \) scans only

\[
N_{L,K} = M_{SC} T_d \; \frac{1}{K} \; \bar{n} \; L \tag{30}
\]

attempts can be made. Seeing as the probability of intercepting a single hop with a multichannel receiver is higher by the factor \( K \) than that of a single-channel receiver \((3, 8)\), the mean number \( \bar{n} \) of hits is the same for the single-channel and the multichannel receiver for the same number \( L \) of scans. The observation time required by the multichannel receiver is shorter by the factor \( 1/K \).
in FIG 8 or (22), assuming that the receiver performs an average of one valid detection attempt per transmitter hop [4].

b) De Moivre-Laplace theorem:
Let the operating time $T_h$ be sufficiently long that, according to (14), $N = 10^4$ is obtained for the number of repeated attempts. With (16, 8 and 7) the mean number of hits is then

$$\bar{k} = N \times P_{1h} = 10000 \times \bar{n}/2000 = 5 \times \bar{n}, \text{where} \quad \bar{n} = (T_h - T_i)/T_d$$

The probability of at least $k$ hits can be approximated using (27). This is shown in FIG 8 for a single-channel receiver and a variable mean number $\bar{n}$ of valid attempts by the receiver during the hop interval $T_h$ (ie with different scan speeds). Trace $\bar{n} = 1$ also applies to a single-channel receiver in wait mode. The same relationship for an eight-channel receiver is plotted in FIG 9. Here the mean number of hits according to (16) with $P = P_{1h}$ (8) is greater by a factor of 8 compared to a single-channel receiver. Trace $\bar{n} = 1$ is also valid for an eight-channel receiver in wait mode. In FIG 10 only one valid detection attempt is assumed per hop interval $[\bar{n} = 1, (T_h - T_i)/T_d = 1 \text{ or wait mode}]$ and the effect of an increased number of parallel receiver channels is shown.

For the probabilities of intercept calculated with (27) and shown in FIGs 8 to 10, the effect of a specific measure (parallel channels, faster scan) can easily be estimated with the aid of the mean number of hits according to (16):

The probability traces $P_N(k)$ reach a value of 0.5 when the minimum achievable number of hits is just equal to the mean number of hits $k = \bar{k}$. With increasing values of $\bar{k}$ the traces consequently shift proportionally to the greater minimum number of hits. Taking a $K$-channel receiver and substituting (8) in (16), the mean number of hits for the coincidence of search and hop frequency range assumed in FIG 5d is given by

$$\bar{k} = N \times P_{1h} = N \frac{K \bar{n}}{M_{FH}}$$

$$= N \frac{K}{M_{FH}} \left( \frac{T_h - T_i}{T_d} \right)$$

(28)

c) Binomial distribution:
If the same relationship as in FIG 10 is to be determined for just a small number of attempts, the binomial distribution must be used without approximations. The diagram shown in FIG 11 is obtained with (19) for only ten repeated attempts. The POI is correspondingly low.

6 Conclusion
Under the conditions described in section 2, the probability of intercept for a single burst (hop) is proportional to the product of the channel number $K$ and the mean number $\bar{n}$ of attempts of a receiver in the hop interval $T_h$ (8, 28). The measures “multichannel receiver” and “fast scan” have the same effect on probability of intercept and are therefore interchangeable. Both measures for increasing probability of intercept require more effort at the receiver end. A search receiver optimized for intercepting sufficiently strong signals works with the largest possible number $K$ of parallel channels and a minimum dwell time $T_d$, ie the shortest possible times for detection, synthesizer settling and signal processing. Short intercept times call for broadband filters. When the detection time $T_i$ is shortened, probability of intercept is limited by reduced selectivity to narrowband adjacent-channel signals and broadband interfering signals. If the interfering signal is broadband noise, the required field strength increases proportionally to $\sqrt{1/T_i}$ for faster detection time $T_i$. If the FH signal can be observed over a specific operating time, the attempts at detection can be repeated and thus the number of hits increased.

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REFERENCES