



Products: R&S® FSMR, R&S® NRP-Z27/37, R&S® NRP-Z51/55

# RF Level Measurement Uncertainties with the Measuring Receiver R&S® FSMR

## Application Note

Measurement uncertainties that occur when the R&S® FSMR measuring receiver is used to calibrate RF levels are described and quantified in this application note. Several Excel spreadsheets are included. They can be used to simplify calculation of the uncertainty for various configurations.



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### **1 Overview**

The R&S® FSMR is an all-in-one system for calibrating signal generators and attenuators. This measuring receiver combines a level calibrator, modulation and audio analyzer, power meter and spectrum analyzer in a single instrument. Due to its high linearity and wide frequency range (20 Hz up to max. 50 GHz, depending on the instrument model), it is ideal for measurement tasks in calibration and test laboratories.

This application note deals with the specific topic of level calibration of signal generators. As a level calibrator, the R&S® FSMR achieves a linearity of 0.015 dB + 0.005 dB/10 dB over a range of +30 dBm to –130 dBm.

Despite the excellent performance features of the R&S® FSMR, level calibration of signal generators (like all measurements) is subject to many different factors that affect the quality of the results. Some examples of sources of uncertainty are mismatches, linearity errors, interference, and temperature effects. To evaluate the measurement uncertainty, it is necessary to quantitatively determine all sources of uncertainty and combine them in a suitable way to obtain the combined uncertainty.

This application note is intended to assist users in that process. It provides explanations and support for determining and minimizing the measurement uncertainty when using the R&S® FSMR for signal generator level calibration. Several Excel spreadsheets are available for downloading. They handle the calculations and so make it easier to quantify the measurement uncertainty.

From this point on, the R&S® FSMR measuring receiver is referred to as the FSMR in this document.

### **2 Measuring principle of the FSMR measuring receiver**

#### **Analog section**

The basic FSMR is a triple-conversion heterodyne receiver that covers the frequency range of 20 Hz to 3.6 GHz (FSMR3). The models for the frequency range of 3.6 GHz to 26.5 GHz (FSMR26) and 3.6 GHz to 50 GHz (FSMR50) use double heterodyne conversion for frequencies above 3.6 GHz.

The design of the mixers and amplifiers used in the analog signal chain combines excellent overload immunity with low noise. As a result, despite its extremely wide level range (–130 dBm to +30 dBm) the FSMR achieves a good signal-to-noise ratio (SNR) without compression effects using only three measurement ranges.

The 3rd IF (2nd IF for frequencies above 3.6 MHz) is sampled by a highly linear 14-bit A/D converter. Its linearity is maintained even at very low signal levels due to special large-scale dithering of the A/D converter. Bandwidth filtering and conversion to the complex baseband (I/Q data) are fully digital.

The accuracy of the A/D converter and the calibration uncertainty are almost solely responsible for the nonlinearity of the FSMR. Modules and

## Level Measurement Uncertainty with the R&S FSMR

components subject to drift (YIG filter) or exhibiting a non-linear level characteristic (crystal filter) are disabled in level measurement mode. The overload-immune analog stages, such as the mixers and IF amplifiers, are operated well below their overload limits to exclude even the smallest compression effects.

To measure standard RF generators over their full range, which typically extends from  $-130$  dBm to  $+10$  dBm, the built-in RF attenuator or IF gain must be switched as necessary. The FSMR eliminates potential level measurement uncertainty resulting from such switching by using adjacent range calibration. At a constant signal level, the measurement result after range switching is corrected to the measurement result before switching. Adjacent range calibration is initiated via the RECAL softkey. A suitable indication (yellow **RECAL** symbol) in the display informs the user that adjacent range calibration is necessary.



Figure 1: A portion of the FSMR display with the recalibration prompt (**RECAL**).

The FSMR also indicates when the signal level is outside the permissible recalibration range by displaying the **UNCORR** indication.



Figure 2: A portion of the FSMR display with the **UNCORR** indication to warn that the level is outside the permissible recalibration range.

## Power ranges

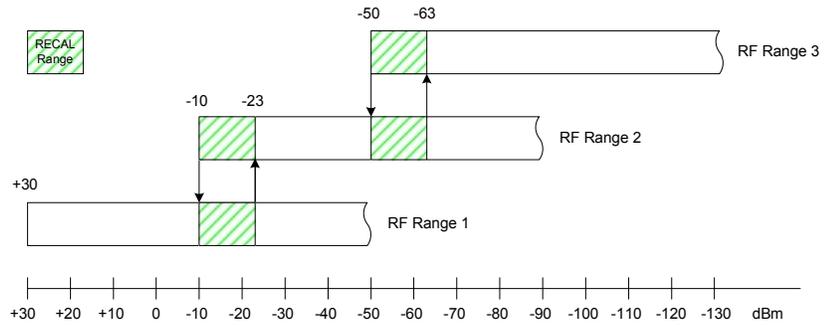


Figure 3: Graphic representation of the FSMR power level ranges

Range	RF ATT	REF LEVEL	PREAMP	Power range
RF Range 1	50 dB	30 dBm	Off	+30 to -20 dBm
RF Range 2	10 dB	-10 dBm	Off	-10 to -60 dBm
RF Range 3	0 dB*)	-50 dBm	On	-50 to -130 dBm

Table 1: FSMR power level ranges

\* The default state of the instrument can be set to 0 dB or 10 dB.

Ranges for adjacent range calibration (RECAL display indication):

Range 1 ↔ Range 2: -10 to -23 dBm

Range 2 ↔ Range 3: -50 to -63 dBm

## Power ranges with the NRP-Z27/-Z-37 power sensor module

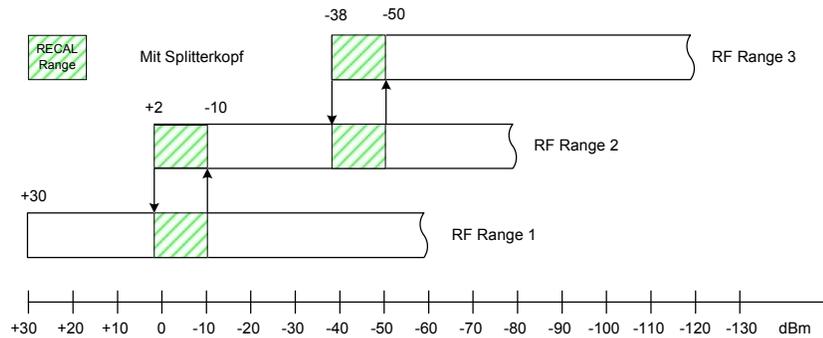


Figure 4: Graphic representation of the FSMR power level ranges with the NRP-Z27/-Z-37 power sensor module

Range	RF ATT	REF LEVEL	PREAMP	Power range
RF Range 1	50 dB	30 dBm	Off	+30 to -10 dBm
RF Range 2	10 dB	0 dBm	Off	+2 to -50 dBm
RF Range 3	0 dB*)	-40 dBm	On	-38 to -120 dBm

Table 2: FSMR power level ranges with the NRP-Z27/-Z-37 power sensor module

\* The default state of the instrument can be set to 0 dB or 10 dB.

RECAL ranges:

Range 1 ↔ Range 2: +2 to -10 dBm

Range 2 ↔ Range 3: -38 to -50 dBm

## Digital processing of readings

### Determining the RF level from the baseband I/Q data

The following section explains the test method and various terms related to digital processing of readings that are important for a clear understanding of the resulting measurement uncertainty, along with potential measurement problems.

### Measuring bandwidth

The measuring bandwidth is equal to the set demodulation bandwidth (DEMODO BW). Signals lying within the measuring bandwidth are measured with no uncertainty restrictions. The measuring bandwidth is 12.5 kHz in the

default state. The FSMR can thus make measurements with a frequency deviation tolerance of  $\pm 6.25$  kHz. The measuring bandwidth (DEMODO BW softkey) can be adjusted over a wide range (100 Hz to 10 MHz) for special measurement tasks, such as using a large bandwidth to measure highly unstable signals from microwave sweep generators.

### **Detection bandwidth**

Two different detection bandwidths can be selected. The narrowband detector is active in the default state. All subsequent uncertainty calculations refer to this detection bandwidth.

### **Wideband detector**

The RMS value is calculated from the I/Q data. Consequently, the detection bandwidth is identical to the measuring bandwidth (DEMODO BW).

### **Narrowband detector**

An FFT is performed on the baseband I/Q data to calculate the RF level within the scanned RF spectrum. The detection bandwidth, and also the **effective noise bandwidth**, is determined by the FFT window function used and the measuring time. That determines the lower measurement limit for RF level measurement. A flat-topped window is used in the FSMR to achieve maximum measurement accuracy. The noise bandwidth of the window is  $3.86/(\text{measuring time})$ , which yields  $3.86/(400 \text{ ms}) = 9.75$  Hz in the default state.

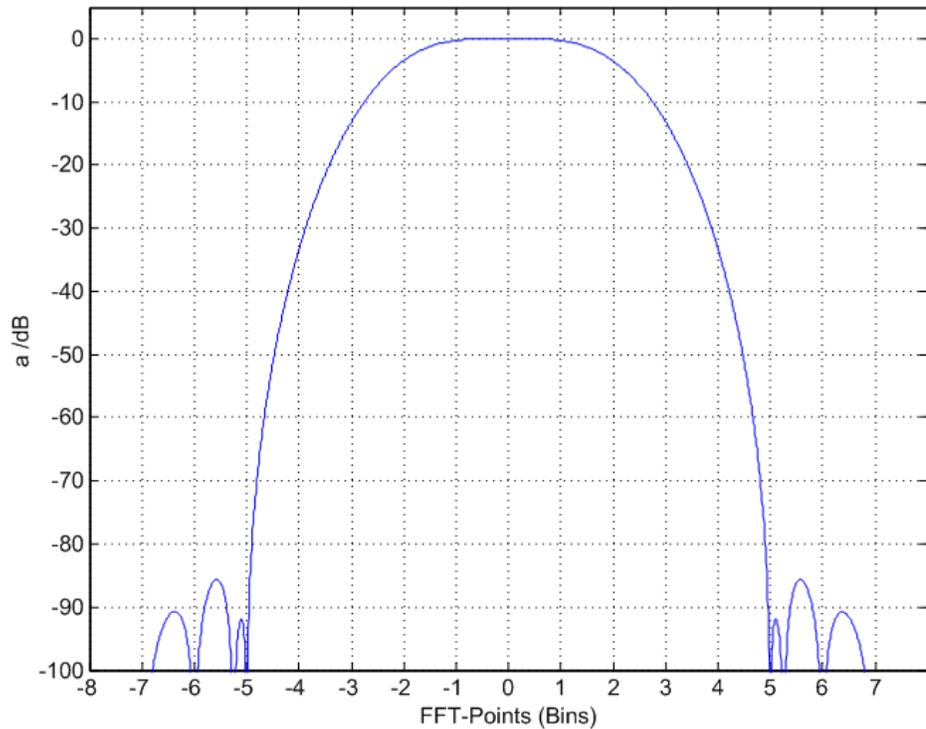


Figure 5: Flat-topped frequency spectrum window. The noise bandwidth is  $3.86/(\text{measuring time}) = 3.86/400 \text{ ms} = 9.75 \text{ Hz}$  in the default state, which is equivalent to 3.88 bins.

### Measuring method using the narrowband detector

The largest signal within the measuring bandwidth is automatically measured (peak search) if the signal-to-noise ratio of the FFT is at least 35 dB. If the signal-to-noise ratio is less than 35 dB, the frequency of the signal detected last is measured. If no signal strong enough to achieve an SNR of 35 dB is present after the receiver frequency is set, the FSMR measures at exactly the set receiver frequency.

The SNR is calculated from the power of the strongest detected signal and the noise power averaged over the measuring bandwidth and normalized to 1 FFT bin.

To prevent noise corruption of the level indicated by the narrowband detector at low SNRs, the noise power is automatically subtracted arithmetically. The resulting indication is thus **noise-corrected**, and statistical variations in the expected value of the test level occur symmetrically.

The FSMR uses the following formula to calculate the standard uncertainty from the measured signal-to-noise ratio and the number of averages n. The calculated value is shown on the display:

$$\text{STD UNC}(A) / dB = \frac{20}{\sqrt{n}} * \log \left( 1 - 10^{-\frac{\text{SNR}}{20}} \right) \quad \text{Formula 1}$$

In Auto Average mode, the FSMR automatically averages the level readings as a function of the measurement level to maintain a low measurement uncertainty due to noise (STD UNC(A)). The dependence is determined by the following table in the default state. This table can be edited to allow the measurement process to be adjusted to suit special requirements. You can thus obtain a smaller variation in the reading due to noise, but at the disadvantage of an increased measuring time (see Formula 1).

AUTO-AVERAGING CONFIGURATION TABLE	
Level range	Number of averages
+30 dBm	1
-70 dBm	2
-80 dBm	4
-90 dBm	8
-100 dBm	16
-110 dBm	32
-120 dBm	64
-130 dBm	128

Table 3: Averaging factor for FSMR Auto Averaging

### 3 Power sensor reference measurement for absolute level measurement accuracy

The highest measurement accuracy for absolute power measurements can be obtained by using a power sensor, such as the thermoelectric NRP-Z51. Due to the insensitivity of thermoelectric measurements, this requires a relatively high power level (typ. >-30 dBm). If you want to measure down to significantly lower power levels (typ. <-70 dBm), even the more sensitive diode power sensors are unsuitable and you are dependent on the selective measuring methods of a measuring receiver. The FSMR measuring receiver offers the highest possible linearity and an extremely wide range extending as low as -130 dBm. However, it cannot achieve the absolute measurement accuracy of a thermoelectric power sensor.

Using an NRP-Z51 thermoelectric power sensor connected to the generator under test, the FSMR can perform a reference measurement at a suitable level (>-30 dBm) with the highest possible accuracy (*Cal Abs Power* function). If you then connect the FSMR RF input (instead of the power sensor) to the generator (screw-connection required) at the same generator level, the FSMR reading can be normalized to the reading previously

## Level Measurement Uncertainty with the R&S FSMR

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obtained using the sensor. The extreme linearity of the FSMR and its wide range then allow you to measure down to  $-130$  dB with the highest absolute accuracy.

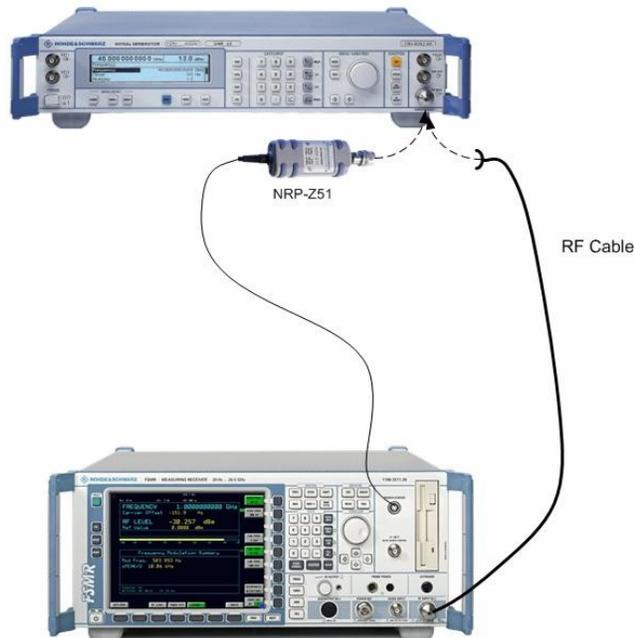


Figure 6: Using an NRP-Z51 power sensor as a reference sensor for high absolute level measurement accuracy with the FSMR

The NRP-Z27/37 power sensor module enables you to combine the advantages of thermoelectric measurements with the reliability and reproducibility of a precision power splitter. The low, constant VSWR and good isolation of the FSMR input significantly reduce measurement uncertainties arising from the source VSWR of the generator under test.

The annoying, time-consuming task of screw-connecting the sensor to the FSMR input is also eliminated with this configuration.

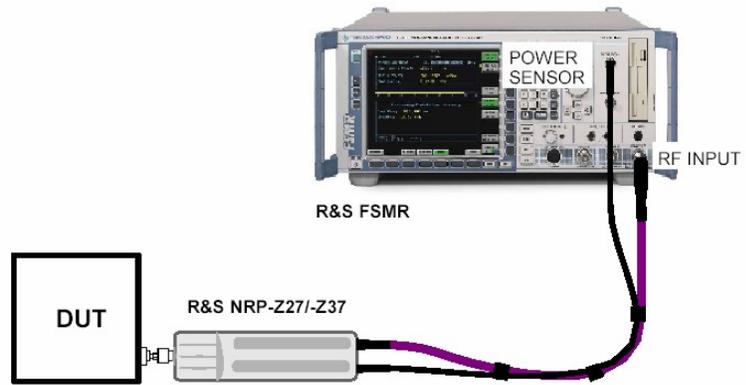


Figure 7: Measuring with the NRP-Z27/37 power sensor module for highest accuracy

## 4 Sources of measurement uncertainty in RF level measurements with the FSMR

The following sources of error must be taken into account when making RF level measurements with the FSMR:

- With absolute power measurements, the **measurement uncertainty in the reference measurement** with the power sensor is the main source of error. The crucial factors here are the sensor specification and the mismatch (VSWR) between the source ( the generator under test) and the sensor. Power sensors have very small VSWRs in comparison with measuring receivers such as the FSMR, so a very small measurement uncertainty can be achieved.
- **Linearity error of the FSMR:** the linearity error is very small compared with other measurement uncertainties.
- **Range-to-range error of the FSMR** (maximum of two range changes). With a matched source, the range-to-range error of the FSMR is very small compared with other measurement uncertainties.
- **Mismatch (VSWR) of the source, cable connection and FSMR** (RF input) if the source VSWR changes. This is normally the case when measuring generator outputs, because the output VSWR of the generator usually depends on the setting of the attenuator at the generator output.

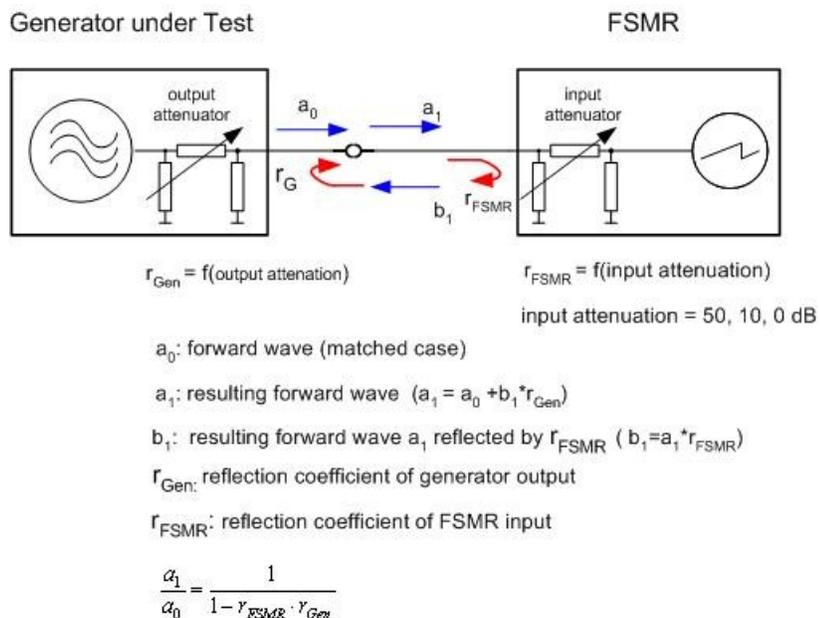


Figure 8: Test scenario for level measurement using the FSMR with imperfect matching

The VSWR of the FSMR, which is relatively large when compared to the VSWR of a power sensor, has a clearly noticeable effect on the combined measurement uncertainty. As the FSMR typically changes the setting of its input attenuator (and thus the VSWR) twice, this plus the changes in the source VSWR result in increased measurement uncertainty due to the VSWR. This measurement uncertainty can be reduced by using a suitable external attenuator with a low VSWR connected ahead of the FSMR RF input, or, better yet, to use an NRP-Z27/Z37 power sensor module. The particular advantage of the NRP-Z27/Z37 power sensor module is its low, constant VSWR, which depends only slightly on the FSMR measurement range due to the high isolation of the module. This reduces additional errors that normally occur with changing load impedance (VSWR of the sensor and the FSMR with 50 dB, 10 dB or 0 dB input attenuation).

- **Statistical measurement uncertainty (Type A uncertainty)** due to the signal-to-noise ratio (SNR) has a crucial effect at low signal levels. It can be reduced by using a longer measuring time and/or a higher averaging factor, which increases the measuring time, however.

## 5 Uncertainty calculations for various configurations

### Absolute power measurement with a power sensor

The error sources of the sensor include the following:

- Calibration uncertainty
- Linearity uncertainty
- Display noise
- Zero offset and drift
- Temperature effects

The next section shows an error calculation for the NRP-Z51 thermoelectric power sensor as an example. Error calculations for other sensors follow the same concept.

Rohde & Schwarz power sensors such as the R&S NRP-Z51 are calibrated in absolute power.

The full range of the NRP-Z51 sensor is  $-30$  dBm to  $+20$  dBm. The data sheet specifies the calibration uncertainty over the  $20$  °C to  $25$  °C temperature range (expanded uncertainty with  $k=2$ ) at the  $0$  dBm calibration level. For other levels, the linearity uncertainty, zero offset and drift, display noise and possibly the temperature drift must be taken into account, in addition to the calibration uncertainty. The uncertainty due to the mismatch between the generator under test and the sensor adds to this.

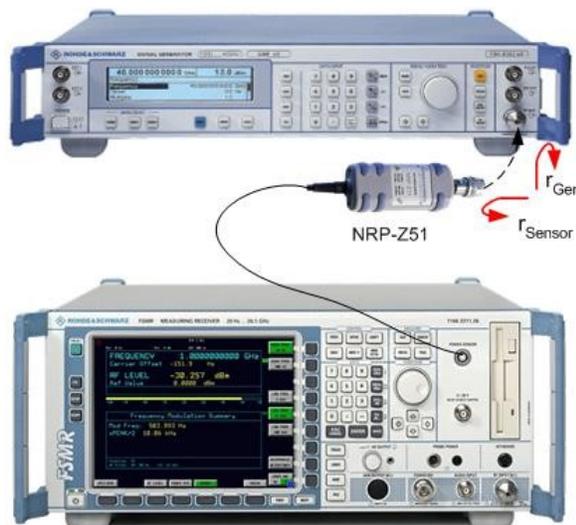


Figure 9: Absolute power measurement using an NRP-Z51 power sensor

### Effect of mismatch

A mismatch between the source (i.e. generator) and the sensor causes additional measurement uncertainty, which in many cases can be quite significant. The reflection factor of a power sensor such as the NRP-Z51 is normally very low compared with that of a typical signal generator. However, because the product of the reflection factors is crucial for the measurement uncertainty, the uncertainty due to the mismatch can make a significant contribution to the combined measurement uncertainty. The following formula is used to calculate the uncertainty:

$$U_{MIS\_Lim} / dB = 20 \log(1 + r_{Gen} \times r_{Sensor}) \quad \text{Formula 2}$$

Using the relationship  $r = \frac{s-1}{s+1}$  Formula 3

we obtain:

$$U_{MIS\_Lim} / dB = 20 \log\left(1 + \frac{s_{Gen} - 1}{s_{Gen} + 1} \times \frac{s_{Sensor} - 1}{s_{Sensor} + 1}\right) \quad \text{Formula 4}$$

where

$U_{Mis\_Lim}$	uncertainty limit due to the mismatch in dB	
$r$	value of the reflection factor	(0 to 1.0)
$r_{Gen}$	value of the reflection factor of the generator	(0 to 1.0)
$r_{Sensor}$	value of the reflection factor of the sensor	(0 to 1.0)
$s$	voltage standing wave ratio (VSWR)	(1.0 to $\infty$ )
$s_{Gen}$	VSWR of the generator	(1.0 to $\infty$ )
$s_{Sensor}$	VSWR of the sensor	(1.0 to $\infty$ )

Due to its U-shaped distribution, the standard uncertainty due to the mismatch ( $U_{MIS\_STB}$ ) is calculated as follows:

$$U_{MIS\_STB} / dB = 20 \log\left(1 + \frac{s_{Gen} - 1}{s_{Gen} + 1} \times \frac{s_{Sensor} - 1}{s_{Sensor} + 1} \times \frac{1}{\sqrt{2}}\right) \quad \text{Formula 5}$$

The following Excel spreadsheet can be used to calculate the uncertainty of the absolute power measurement for each level and each frequency over the specified temperature range of 20 °C to 25 °C. The corresponding values for the frequency range of 100 MHz to 4 GHz ("Data" column in the Excel spreadsheet) are taken from the data sheet.

For other levels or frequencies, the corresponding values must be copied from the data sheet to the Excel spreadsheet.

To calculate the standard uncertainty of the individual components, the spreadsheet takes into account the type of distribution exhibited by the specific parameters affecting the uncertainty and whether they are expressed as an expanded uncertainty ( $k = 2$ ) or a standard uncertainty. Logically enough, the standard uncertainty is calculated by dividing the expanded uncertainty ( $k = 2$ ) by 2.

If the maximum value is indicated and there is a uniform distribution of the measurement error between the limits, such as is the case for the error due to temperature, the standard uncertainty is the maximum value in dB divided by  $\sqrt{3}$  (see Formula 5).

With a U-shaped distribution, which is applicable to mismatch errors, the standard uncertainty is the value in dB divided by  $\sqrt{2}$ .

### Calculating the combined standard uncertainty

To calculate the combined standard uncertainty of the power measurement, the squares of the individual standard uncertainties are first summed. The square root of the sum of the squares then yields the combined standard uncertainty of the power measurement.

This is then multiplied by the factor  $k (= 2)$  to obtain the expanded uncertainty.

### How to use the Excel spreadsheets

Calculating the measurement uncertainty with the Excel spreadsheets is quite easy. Simply enter the level to be measured in the "Data" column and the sensor data corresponding to the measurement frequency in the cells highlighted in yellow (as well as the corresponding FSMR data in the following Excel spreadsheets). You will find this data in the data sheet of the sensor (or the FSMR). All calculations are performed automatically.

Once you have entered all the data, you can read the combined measurement uncertainty from the row highlighted in green at the bottom of the table. The values for the frequency range of 100 MHz to 4 GHz have been entered in the spreadsheet as an example. The test level is taken to be 0 dBm. The calculated value of the expanded uncertainty ( $k = 2$ ) is 0.183 dB in the default state.

To take another example, if you want to determine the measurement uncertainty when testing a generator at 10 GHz at a test level of  $-10$  dBm, the only changes required are as follows:

Test level:  $-10$  dBm

Calibration uncertainty: 0.076 dB (from the NRP-Z51 data sheet)

(The specified VSWR of the generator under test is assumed to be 1.5)

The expanded uncertainty with the changed values is 0.190 dB.

**Note:** Only change the Excel spreadsheet cells that are highlighted in yellow. All other cells are protected to prevent accidental modification of the formulas. This protection is not password-protected, so it can be removed if necessary. That allows you to view the formulas used for the calculations.

## Level Measurement Uncertainty with the R&S FSMR

You can save the modified spreadsheets, but that will overwrite the original Excel spreadsheets. For that reason, we recommend that you save a copy of the original data.

	A	B	C	D	E	F
1	<b>FSMR Absolute Power Measurement Uncertainty with NRP-Z51 100 MHz to 4 GHz (Powermeter Measurement):</b>					
2		<b>Distribution function</b>	<b>Data</b>	<b>Calcul. interim results</b>	<b>Unit</b>	<b>Standard Uncertainty in dB (1 <math>\sigma</math>).</b>
3	<b>Test Level in dBm</b>		0	1,000E-03	W	
4	Specification of Display noise in nW (10,24s integration time)		30			
5	Display noise (1 s integration time )	gaussian (k=2 )		9,60E-08	W	
6	Standard Uncertainty					0,000
8	Zero Offset in nW	gaussian (k=2)	50			
9	Standard Uncertainty					0,000
11	Zero Drift in nW	gaussian (k=2)	20			
12	Standard Uncertainty					0,000
14	Calibration uncertainty in dB (20 to 25 °C)	gaussian (k=2)	0,057			
15	Standard Uncertainty					0,029
17	Linearity Error in dB	gaussian (k=2)	0,02			
18	Standard Uncertainty					0,010
21	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor and Matched Source (in dB)</b>					<b>0,030</b>
23	<b>Uncertainty due to mismatch</b>					
24	VSWR <sub>Sensor</sub>		1,15	0,0698	--	
25	VSWR <sub>Generator</sub>		1,5	0,2000	--	
26	<b>Standard Uncertainty</b>					<b>0,086</b>
27	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor in dB</b>					<b>0,091</b>
28	<b>Expanded Uncertainty of Power Measurement with Power Sensor in dB ( k=2)</b>					<b>0,183</b>

Spreadsheet 1: Calculating the measurement uncertainty for absolute power measurements using an R&S NRP-Z51 power sensor

### Relative level measurement with the FSMR

Of course, you can also use the FSMR to measure power levels relative to a reference level instead of absolute power. Measurement using the power sensor is omitted in this case. Calibrating step attenuators is an example of a typical application for relative level measurements.

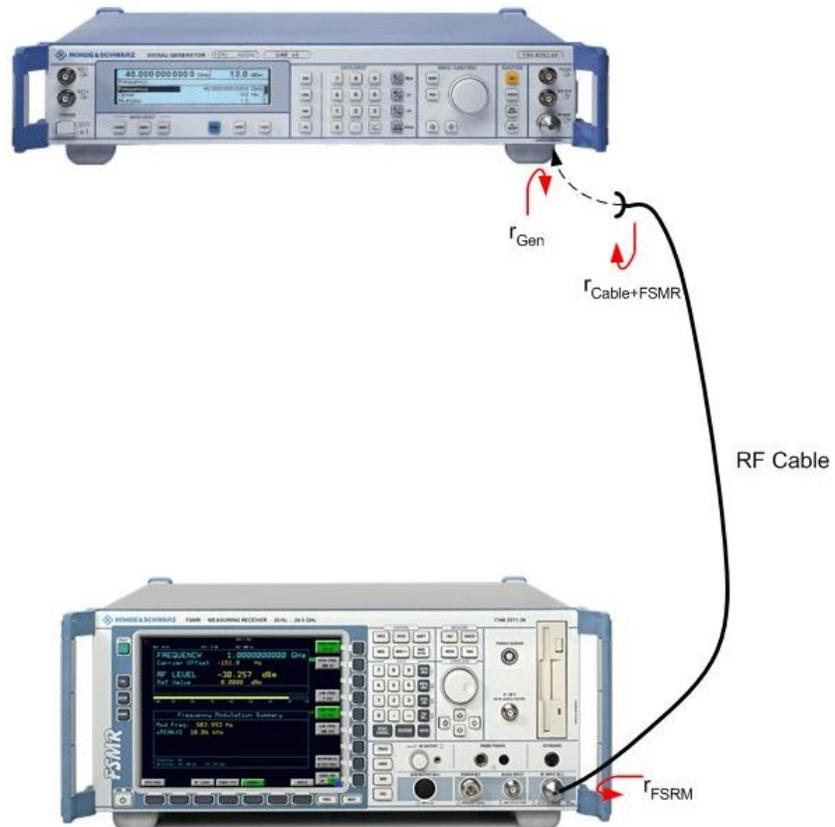


Figure 10: Test setup for relative level measurements using the FSMR

The following spreadsheet is used to calculate the measurement uncertainty for this case.

The data to be entered is:

- The **reference level** and the **level to be measured**.
- The specified **range-to-range error** of the FSMR (0.005 dB for frequencies up to 22 GHz) depending on the frequency range.
- The **VSWR** of the generator (DUT) and the FSMR (for 0 dB or 50/10 dB attenuation) depending on the test frequency and test level. The Excel spreadsheet takes into account the effect of FSMR range switching, or more precisely attenuator switching, on the measurement uncertainty due to mismatch. It also takes into account the fact that the 10 dB attenuator is directly connected to the FSMR output and is permanently on (except for 0 dB attenuation). The adjacent range calibrations are made at -20 dBm and at -60 dBm. For the sake of simplicity, the effect

## Level Measurement Uncertainty with the R&S FSMR

of the test cable is neglected in the calculation. That is acceptable when a precision test cable with a correspondingly low VSWR (<1.2) is used. If several mutually independent components are present and they all affect the measurement uncertainty due to mismatch, the square root of the sum of the squares of the individual components (combined standard uncertainty) is derived.

- The specified **DANL** (displayed average noise level) of the FSMR, normalized to a 10 Hz resolution bandwidth (RBW). The typical DANL of the FSMR 26 at 4 GHz (-144 dBm) is used the following example.
- The set **measuring time of the FSMR** (400 ms in the default state).
- The **averaging number** of the FSMR. In the default state, it is set automatically to match the level. The standard deviation of the Type A measurement uncertainty is calculated from the DANL, the measuring time and the averaging number, and it is then used in the Excel spreadsheet. Alternatively, you can enter the actual Type A uncertainty measured and displayed by the FSMR (STD UNCERT(A) on the FSMR display) instead of the N.A. (not available) entry. The Excel spreadsheet is programmed in such a way that a numeric value in this cell takes precedence over the calculated value and is used to calculate the combined measurement uncertainty.

	A	B	C	D	E	F
1	<b>FSMR Relative Level Measurement Uncertainty</b>					
2	(device under test changes VSWR with level change within specification)					
3		<b>Distribution function</b>	<b>Data</b>	<b>Calcul. interim results</b>	<b>Unit</b>	<b>Standard Uncertainty in dB</b>
4	<b>Reference Level in dBm</b>		0	1,000E-03	W	
5	Relative Level to measure/dB		-110			
6	Range to Range Error/dB		0,005			
7						
8	<b>Uncertainty due to Linearity</b>					
9	Calculated Dynamic range			110	dB	
10	Calculated linearity error limit at level to measure	gaussian ( k=2 )		0,075	dB	
11	Resultant standard uncertainty due to linearity					0,038
12						
13	<b>Mismatch Generator Output - FSMR input</b>	U-shaped				
14	min. FSMR attenuation (10 dB or 0 db )/dB		0	1,000		
15	VSWR Generator		1,5	0,200	--	
16	VSWR FSMR 50/10 dB attenuation		1,3	0,130	--	
17	VSWR FSMR 0 dB attenuation		2	0,333	--	
18	Standard Uncertainty incl. Range Switching	1 Range (50dB)		0,224		
19		2 Ranges (50/10dB)				
20		or 3 Ranges (50/10/10 dB)		0,225		
21	<b>Type A Uncertainty due to S/N (calculated)</b>	3 Ranges (50/10/0dB)		0,492		
22	Typ. Displayed Average Noise Level (normalized to 10 Hz) in dBm		-144			0,492
23	MeasTime/ms		400			
24	Number of AVG		64			
25	Calculated NoiseBW			9,75	Hz	
26	Calculated S/N			31,61	dB	
27	Calculated standard uncertainty due to S/N and AVG			0,0289	dB	
28	FSMR's Displayed Meas. Type A Uncertainty due to S/N and AVG in dB		N.A.			
29	Standard uncertainty due to S/N in dB used for Combined Uncertainty calculation					0,029
30						
31	<b>Combined Standard Uncertainty of Relative Level Measurement in dB</b>					
32	<b>Expanded Uncertainty of Relative Level Measurement in dB (k=2)</b>					
						<b>0,988</b>

Spreadsheet 2: Calculating the relative measurement uncertainty of the FSMR

As you can see, the measurement uncertainty due to the VSWR represents the main component of the combined measurement uncertainty. By contrast, errors due to the nonlinearity of the FSMR and Type A uncertainty do not contribute significantly to the combined uncertainty.

### Reducing the measurement uncertainty due to the VSWR

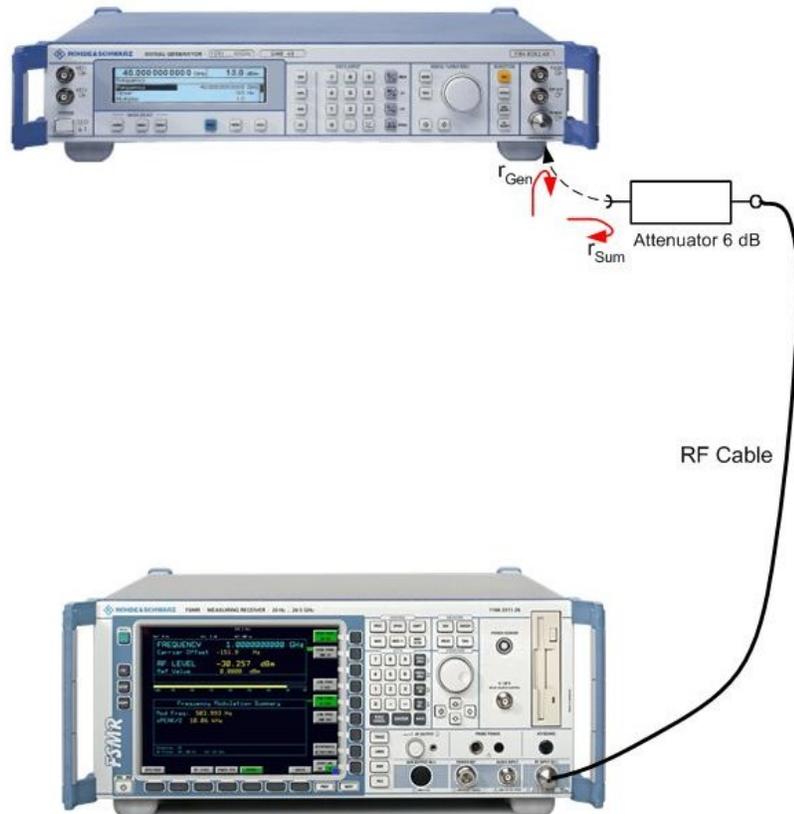


Figure 11: Test setup for relative level measurement using the FSMR with an attenuator connected ahead.

The VSWR of the FSMR and the resulting measurement uncertainty can be significantly reduced by connecting a suitable precision attenuator, e.g. with 6 dB or 10 dB attenuation, ahead of the FSMR. However, that also reduces the test level at the FSMR input and thus the signal-to-noise ratio, which in turn increases the Type A measurement uncertainty. A more effective approach is to use an external attenuator instead of the internal 10 dB FSMR step attenuator setting (FSMR function: *Min. Attenuation 10 dB*), as considerably lower VSWR values can be achieved with this configuration (typ. 1.1 or less at frequencies up to several GHz). It is advantageous to attach the attenuator directly to the generator under test rather than connecting it after the test cable. With a high-quality test cable (VSWR <1.2), the cable can then be neglected in the calculation because the VSWR of the FSMR will be the dominant factor.

The Excel spreadsheet corresponding to the above test setup using an attenuator is shown below. Besides the data for Table 2, the **attenuation of the external attenuator** (10 dB in the original version of the Excel spreadsheet) and the **specified (or measured) VSWR** of the attenuator (1.1 in the original version of the Excel spreadsheet) must also be entered in this case.

## Level Measurement Uncertainty with the R&S FSMR

	A	B	C	D	E	F
1	<b>FSMR Relative Level Measurement Uncertainty (attenuator in front of FSMR input):</b>					
2		<b>Distribution function</b>	<b>Data</b>	<b>Calcul. interim results</b>	<b>Unit</b>	<b>Standard Uncertainty in dB</b>
3	<b>Reference Level in dBm</b>		<b>0</b>	1,000E-03	W	
4	Relative level to measure/dB		<b>-110</b>			
5	Range to Range Error/dB		<b>0,005</b>			
6						
7	<b>Uncertainty due to Linearity</b>					
8	Calculated Dynamic range			110	dB	
9	Calculated linearity error at level to measure	gaussian (k=2)		0,075	dB	
10	Calculated Standard deviation of linearity error					<b>0,038</b>
11						
12	<b>Mismatch Generator Output - FSMR input</b>	U-shaped				
13	min. FSMR attenuation (10 dB or 0 db )/dB		<b>0</b>	1,0000		
14	Attenuation of attenuator in front of FSMR input		<b>10</b>	0,3162	--	
15	VSWR generator		<b>1,5</b>	0,2000		
16	VSWR attenuator		<b>1,1</b>	0,0476	--	
17	VSWR FSMR 50/10 dB attenuation		<b>1,3</b>	0,1304	--	
18	VSWR FSMR 0 dB attenuation		<b>2</b>	0,3333		
19	Standard Uncertainty incl. Range Switching			0,0127	--	
20		1 Range (50dB)		0,085		
21		2 Ranges (50/10dB)				
22		or 3 Ranges (50/10/10 dB)		0,088		
23		3 Ranges (50/10/0dB)		0,105		
24	<b>Type A Uncertainty due to S/N (calculated)</b>					<b>0,105</b>
25	Typ. Displayed average Noise Level (normalized to 10 Hz) in dBm		<b>-144</b>			
26	MeasTime/ms		<b>400</b>			
27	Number of AVG		<b>64</b>			
28	Calculated NoiseBW			9,75	Hz	
29	Calculated S/N			21,61	dB	
30	Calculated standard uncertainty due to S/N and AVG			0,094	dB	
31	Displayed Meas. Type A Uncertainty due to S/N and AVG in dB		<b>N.A.</b>			
32	Standard uncertainty due to S/N in dB used for Combined Uncertainty calculation					<b>0,094</b>
33						
34	<b>Combined Standard Uncertainty of Relative Level Measurement in dB</b>					<b>0,146</b>
35	<b>Expanded Combined Uncertainty of Relative Level Measurement in dB ( k=2)</b>			<b>0,292</b>		

Spreadsheet 3: Calculating the relative measurement uncertainty of the FSMR with an upstream attenuator

In the case of the selected example, the standard deviation of the measurement uncertainty due to mismatch is reduced from 0.49 dB (spreadsheet 2) to 0.1 dB (spreadsheet 3) by virtue of the attenuator connected ahead. However, the Type A measurement uncertainty increases slightly because the attenuator reduces the level at the FSMR input in proportion to its attenuation, which reduces the signal-to-noise ratio. However, this can be countered by increasing the averaging factor (by a factor of 4 for a 6 dB attenuator) if a longer measuring time is acceptable. Overall, the expanded combined standard uncertainty is reduced from 0.99 dB (spreadsheet 2) to 0.29 dB (spreadsheet 3) with the same measuring time.

### Absolute RF power measurements at low levels

For making power measurements with the FSMR outside the range of the power sensor, such as below  $-20\text{dB}$  with an NRP-Z51, you can normalize the measuring receiver to a reference level (e.g.  $0\text{ dBm}$ ) on the power sensor display (using the CAL ABS Power softkey). In measuring receiver mode, the FSMR will then display the reference power with the same accuracy as previously obtained using the power sensor. This normalization initially eliminates the additional error due to the relatively high VSWR of the FSMR RF input (compared with the power sensor).

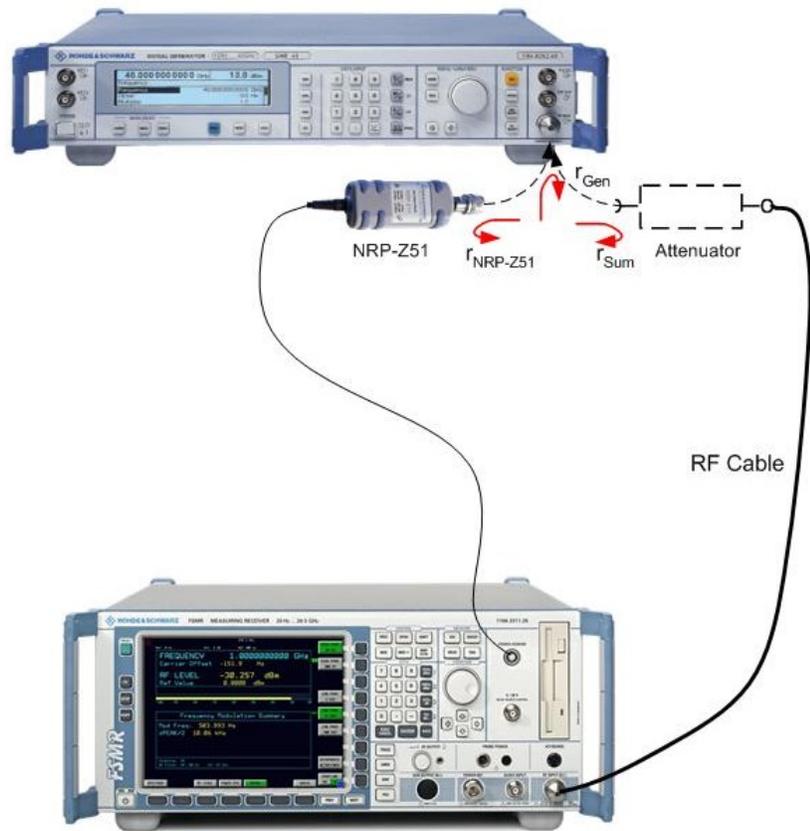


Figure 12: Test setup for power measurements with the FSMR and an NRP-Z51 sensor (or external attenuator as necessary)

The VSWR of the generator changes when the generator power is reduced, which usually occurs when the step attenuator at the generator output switches. In that case there is an additional error that depends on the VSWR of the FSMR and the generator. To keep this error small for absolute power measurements with the FSMR, it is also always advisable to use an external attenuator ahead of the FSMR test cable.

The following two spreadsheets show sample uncertainty calculations for absolute level measurements down to  $-110\text{ dBm}$  with and without an attenuator connected ahead.

## Level Measurement Uncertainty with the R&S FSMR

	A	B	C	D	E	F
1	<b>FSMR Absolute Power Measurement Uncertainty 100 MHz to 4 GHz:</b>					
2	<b>(Reference Measurement with NRP-Z51)</b>					
3		<b>Distribution function</b>	<b>Data</b>	<b>Calcul. interim results</b>	<b>Unit</b>	<b>Standard Uncertainty in dB (1 <math>\sigma</math>).</b>
4	<b>Reference Level in dBm</b>		<b>0</b>	1,000E-03	W	
5	Specification of Display noise in nW (10,24s integration time)		<b>30</b>			
6	Display noise (1 s integration time)	gaussian (k=2)		9,60E-08	W	
7	Standard Uncertainty					0,000
8						
9	Zero Offset in nW	gaussian (k=2)	<b>50</b>			
10	Standard Uncertainty					0,000
11						
12	Zero Drift in nW	gaussian (k=2)	<b>20</b>			
13	Standard Uncertainty					0,000
14						
15	Calibration uncertainty in dB (20 to 25 °C)	gaussian (k=2)	<b>0,057</b>			
16	Standard Uncertainty					0,029
17						
18	Linearity Error in dB	gaussian (k=2)	<b>0,02</b>			
19	Standard Uncertainty					0,010
20						
21						
22	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor and Matched Source (in dB)</b>					<b>0,030</b>
23						
24	<b>Uncertainty due to mismatch of power sensor</b>					
25	VSWR <sub>Sensor</sub>		<b>1,15</b>	0,0698	--	
26	VSWR <sub>Generator</sub>		<b>1,5</b>	0,2000	--	
27	Standard Uncertainty					<b>0,0861</b>
28	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor in dB</b>					<b>0,091</b>
29						
30	<b>FSMR Relative Level Measurement Uncertainty:</b>					
31						
32	<b>Level to measure/dBm</b>		<b>-110</b>			
33	Range to Range Error/dB		<b>0,005</b>			
34						
35	<b>Uncertainty due to Linearity</b>					
36	Calculated Dynamic range			110	dB	
37	Calculated linearity error at level to measure	gaussian (k=2)		0,065	dB	
38	Calculated Standard Uncertainty due to linearity error					<b>0,033</b>
39						
40	<b>Uncertainty due to Mismatch</b>					
41	min. FSMR attenuation (10 dB or 0 db )/dB	U-shaped	<b>0</b>	1,0000		
42	VSWR Generator		<b>1,5</b>	0,2000		
43	VSWR FSMR 50/10 dB attenuation		<b>1,3</b>	0,1304	--	
44	VSWR FSMR 0 dB attenuation		<b>2</b>	0,3333		
45						
46				0,224	dB	
47						
48				0,225	dB	
49	Calculated Standard Uncertainty			0,492	dB	
50						<b>0,492</b>
51	<b>Type A Uncertainty due to S/N</b>					
52	Typ. Displayed average Noise Level (normalized to 10 Hz) in dBm		<b>-144</b>			
53	MeasTime/ms		<b>400</b>			
54	Number of AVG		<b>64</b>			
55	Calculated NoiseBW			9,75	Hz	
56	Calculated S/N			31,61	dB	
57	Calculated standard uncertainty due to S/N and AVG			0,029	dB	
58	Displayed Meas. Type A Uncertainty due to S/N and AVG in dB		<b>N.A.</b>			
59	Standard uncertainty due to S/N in dB used for Combined Uncertainty calculation					<b>0,029</b>
60						
61	<b>Combined Standard Uncertainty of Absolute Level Measurement in dB</b>					<b>0,502</b>
62	<b>Expanded Combined Uncertainty of Absolute Level Measurement in dB ( k=2)</b>				<b>1,004</b>	

Spreadsheet 4: Calculating the measurement uncertainty of the FSMR with an NRP-Z51 power sensor for absolute power measurement (without an attenuator connected ahead)

## Level Measurement Uncertainty with the R&S FSMR

	A	B	C	D	E	F
1	<b>FSMR Absolute Power Measurement Uncertainty 100 MHz to 4 GHz:</b>					
2	<b>(Reference Measurement with NRP-Z51, Attenuator in Front of FSMR RF Input)</b>					
		<b>Distribution function</b>	<b>Data</b>	<b>Calcul. interim results</b>	<b>Unit</b>	<b>Standard Uncertainty in dB (1 <math>\sigma</math>).</b>
3						
4	<b>Reference Level in dBm</b>		<b>0</b>	1,000E-03	W	
5	Specification of Display noise in nW (10,24s integration time)		<b>30</b>			
6	Display noise (1 s integration time)	gaussian (k=2)		9,60E-08	W	
7	Standard Uncertainty					0,000
8						
9	Zero Offset in nW	gaussian (k=2)	<b>50</b>			
10	Standard Uncertainty					0,000
11						
12	Zero Drift in nW	gaussian (k=2)	<b>20</b>			
13	Standard Uncertainty					0,000
14						
15	Calibration uncertainty in dB (20 to 25 °C)	gaussian (k=2)	<b>0,057</b>			
16	Standard Uncertainty					0,029
17						
18	Linearity Error in dB	gaussian (k=2)	<b>0,02</b>			
19	Standard Uncertainty					0,010
20						
21						
22	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor and Matched Source (in dB)</b>					
23						<b>0,030</b>
24	<b>Uncertainty due to mismatch of power sensor</b>					
25	VSWR <sub>Sensor</sub>		<b>1,15</b>	0,0698	--	
26	VSWR <sub>Generator</sub>		<b>1,5</b>	0,2000	--	
27	Standard Uncertainty					<b>0,0861</b>
28	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor in dB</b>					
29						<b>0,091</b>
30	<b>FSMR Relative Level Measurement Uncertainty:</b>					
31						
32	<b>Level to measure/dBm</b>		<b>-110</b>			
33	Range to Range Error/dB		<b>0,005</b>			
34						
35	<b>Uncertainty due to Linearity</b>					
36	Calculated Dynamic range			110	dB	
37	Calculated linearity error at level to measure	gaussian (k=2)		0,065	dB	
38	Calculated Standard Uncertainty due to linearity error					<b>0,033</b>
39						
40	<b>Uncertainty due to Mismatch</b>					
41	min. FSMR attenuation (10 dB or 0 dB )/dB	U-shaped	<b>0</b>	1,0000		
42	Attenuation of attenuator in front of FSMR input		<b>10</b>	0,3162	--	
43	VSWR Generator		<b>1,5</b>	0,2000		
44	VSWR attenuator		<b>1,1</b>	0,0476	--	
45	VSWR FSMR 50/10 dB attenuation		<b>1,3</b>	0,1304	--	
46	VSWR FSMR 0 dB attenuation		<b>2</b>	0,3333		
47						
48						
49				0,085	dB	
50				0,088	dB	
51	Calculated Standard Uncertainty			0,105	dB	
52						<b>0,105</b>
53	<b>Type A Uncertainty due to S/N</b>					
54	Typ. Displayed average Noise Level (normalized to 10 Hz) in dBm		<b>-144</b>			
55	MeasTime/ms		<b>400</b>			
56	Number of AVG		<b>64</b>			
57	Calculated NoiseBW			9,75	Hz	
58	Calculated S/N			21,61	dB	
59	Calculated standard uncertainty due to S/N and AVG			0,094	dB	
60	Displayed Meas. Type A Uncertainty due to S/N and AVG in dB		<b>N.A.</b>			
61	Standard uncertainty due to S/N in dB used for Combined Uncertainty calculation					<b>0,094</b>
62						
63	<b>Combined Standard Uncertainty of Absolute Level Measurement in dB</b>					
64	<b>Expanded Combined Uncertainty of Absolute Level Measurement in dB ( k=2)</b>					
						<b>0,343</b>

Spreadsheet 5: Calculating the measurement uncertainty of the FSMR with an NRP-Z51 power sensor for absolute power measurement (with an attenuator connected ahead of the FSMR input)

As before, using an external attenuator also yields a significantly lower combined measurement uncertainty for absolute power measurements. In the examples shown here, it is 0.34 dB with an attenuator versus 1 dB without an attenuator.

### Measurement with the NRP-Z27/-Z37 power sensor module



Figure 13: Test setup with an NRP-Z27 power sensor module

The power sensor module allows measurements to be made in parallel using a power sensor and the FSMR. A resistive power splitter divides the power between the power sensor and the FSMR receiver input, which is additionally decoupled by a 6 dB attenuator. A precision RF cable connects the FSMR receiver input to the appropriate output (after the 6 dB attenuator) of the power sensor module. The residual effect of the FSMR input on the sensor is corrected numerically. The advantage of this setup is that the time required for the test is reduced considerably because the time-consuming task of disconnecting and reconnecting the cables between measurements is eliminated. In addition, calibration using the power sensor can be automated.

However, the insertion loss of the power sensor module (typ. 13 dB, max. 15 dB at 4 GHz) reduces the input sensitivity of the FSMR accordingly, so that it may be advisable to make measurements without the power sensor module at very low power levels (less than approx.  $-120$  dBm) if a preamplifier is not used.

The measurement uncertainty of the power sensor module is calibrated in absolute power with the FSMR connected. That eliminates the otherwise complex process of taking the power splitter into account in the uncertainty analysis.

Once again, the uncertainty for measurements using the power sensor can be calculated quite easily using the spreadsheets shown below.

In contrast to spreadsheet 5, the input data for the attenuator is omitted, but the following additional input data is required:

- The **VSWR of the power sensor module** at the test frequency (1.18 at 4 GHz).
- The **isolation to the FSMR input** (numerically corrected or uncorrected value, as applicable). In the example, 40 dB is used as the mean corrected value.
- The **insertion loss (attenuation) of the power sensor module** to the FSMR input at the test frequency (typ. 13 dB at 4 GHz).

## Level Measurement Uncertainty with the R&S FSMR

	A	B	C	D	E	F
1	<b>FSMR absolut level measurement uncertainty with NRP-Z27 power sensor module 100 MHz to 4 GHz</b>					
2		<b>Distribution</b>	<b>Data</b>	<b>Value</b>	<b>Unit</b>	<b>Standard</b>
3	<b>Power Sensor Measurement</b>					
4	<b>Test Level in dBm</b>		0	1,000E-03	W	
5	Specification of Display noise in nW (10,24s integration time)		240			
6	Display noise (1 s integration time )	gaussian (k=2)		7,68E-07	W	
7	Standard Uncertainty					0,002
8						
9	Zero Offset in nW	gaussian (k=2)	400			
10	Standard Uncertainty					0,001
11						
12	Zero Drift in nW	gaussian (k=2)	160			
13	Standard Uncertainty					0,000
14						
15	Calibration uncertainty in dB (20 to 25 °C)	gaussian (k=2)	0,07			
16	Standard Uncertainty					0,035
17						
18	Linearity Error in dB	gaussian (k=2)	0,02			
19	Standard Uncertainty					0,010
20	<b>Combined Standard Uncertainty of Power Measurement with Power Sensor and Matched Source (in dB)</b>					<b>0,036</b>
21						
22	<b>Uncertainty due to mismatch</b>					
23	VSWR of FSMR Input at 50/10 dB attenuation		1,3	0,13		
24	VSWR of FSMR Input at 0 dB attenuation		2	0,33		
25	Isolation of FSMR-Input (uncorrected or corrected value whatever applies)		40	0,010		
26	VSWR <sub>Sensor Module</sub>		1,18	0,0826	--	
27	VSWR <sub>Generator</sub>		1,5	0,2000	--	
28	<b>Calculated Standard Uncertainty due to Mismatch at Reference Measurement</b>					<b>0,102</b>
29						
30	<b>Combined Standard Uncertainty of Reference Power Measurement with Power Sensor in dB</b>					<b>0,109</b>
31						
32	<b>FSMR relative level measurement:</b>					
33	Level to measure/dBm		-110			
34	Range to Range Error/dB		0,005			
35	<b>Uncertainty due to Linearity</b>					
36	Resultant Dynamic range			110	dB	
37	Resultant linearity error at level to measure	gaussian (2σ)		0,075	dB	
38	Resultant Standard deviation of linearity error					0,038
39						
40	<b>Uncertainty due to mismatch change of FSMR (for absolute indication):</b>					
41	(condition: output impedance of DUT changes)					
42	Minimum FSMR Attenuation (0dB or 10 dB)		0			
43	Standard Uncertainty due to mismatch change/dB					
44						
45		1 Range (50dB)		0,000		
46		2 Ranges 50/10dB				
47		or 3 Ranges 50/10/10 dB		0,002		
48		3 Ranges 50/10/0 dB		0,013		0,013
49	<b>Type A Uncertainty due to S/N (calculated)</b>					
50	Typ. Displayed average Noise Level (normalized to 10 Hz) in dBm		-144			
51	Attenuation NRP-Z27Input to FSMR Input		13			
52	MeasTime/ms		400			
53	Number of AVG		64			
54	Resultant NoiseBW			9,75	Hz	
55	Resultant S/N			18,61	dB	
56	Calculated standard uncertainty due to S/N and AVG			0,128	dB	
57	Displayed Meas. Type A Uncertainty due to S/N and AVG in dB		N.A.			
58	Standard uncertainty due to S/N in dB used for Combined Uncertainty calculation					0,128
59						
60	<b>Combined Standard Uncertainty of Absolute Level Measurement in dB</b>					<b>0,173</b>
61	<b>Expanded Uncertainty of Absolute Level Measurement in dB (k=2)</b>					<b>0,345</b>

Spreadsheet 6: Calculating the absolute measurement uncertainty with an NRP-Z27/-Z37 power sensor module

### Summary:

With an NRP-Z27/37 power sensor module, the FSMR achieves the same low measurement uncertainty of 0.34 dB (with numeric correction for isolation) as with an NRP-Z51 power sensor and a precision attenuator. The particular advantage of this configuration is that the measurements can be automated because the time-consuming process of connecting and disconnecting test cables, which also causes material wear, is eliminated.

### **Using a preamplifier to reduce the measurement uncertainty due to the SNR**

A preamplifier (FSU-B25 or FSMR-B23) can reduce the input noise of the FSMR and thus significantly improve the SNR at low input powers. As a result, the DANL of the FSMR26 at 4 GHz can typically be reduced from -144 dBm to -153 dBm, which is a difference of approximately 9 dBm. Compared with making measurements without a preamplifier, this makes it possible to measure levels that are around 10 dB weaker with nearly the same measurement uncertainty and measuring time.

Excel spreadsheets 2 to 6 are also suitable for determining the measurement uncertainty with a test setup using a preamplifier. In that case, you simply have to replace the number for the displayed average noise level with the corresponding value specified for the selected preamplifier option (such as -153 dBm at a test frequency of 4 GHz with the FSMR-B23 option). The specified VSWR values also apply when a preamplifier is used.

### **Avoiding measurement uncertainty due to residual generator FM and frequency drift**

In the default state with a set measuring time of 400 ms, the bandwidth of the narrowband detector is approximately 10 Hz (3.86/400 ms). This value is also used as a default in the Excel spreadsheets. However, in the case of generators with strong residual FM, especially microwave generators, a measuring bandwidth of approximately 10 Hz can be too narrow to capture the full signal power. An indication of this is a fluctuating level indication on the FSMR, with a variation exceeding 0.1 dB even for the reference level measurement.

In that case, you can take advantage of the fact that the bandwidth of the narrowband detector depends on the measuring time. Halving the measuring time (*MEAS TIME MANUAL* in the ***SWEEP*** menu) doubles the bandwidth of the narrowband detector. If you enter a shorter measuring time, the bandwidth of the narrowband detector automatically increases. It may be possible to capture the full signal power by using this form of bandwidth adjustment.

A drawback of this approach is that the increased bandwidth reduces the signal-to-noise ratio, resulting in higher Type A uncertainty. However, this can be countered by increasing the averaging factor. When the measuring time is halved, doubling the averaging factor compensates the increased Type A uncertainty.

The FSMR tolerates frequency errors and tracks the measured signal as long as it is inside the measuring bandwidth (Demod BW:  $\pm 6.25$  kHz in the default state). Frequency errors and slow frequency drift of the generator under test thus have no effect on the level measurement accuracy of the FSMR.

**Note:** At low signal powers (less than approximately -105 dBm) or when the SNR measured by the FSMR is less than 35 dB, the FSMR remains at the last measured frequency. In that case, the signal must be free of frequency drift to avoid compromising the level measurement accuracy. It is thus advisable to use the reference frequency to synchronize the generator to the FSMR to avoid frequency drift during the measurement.

## 6 Explanation of terms and glossary

The terms listed below are used in this application note.

### Standard deviation

The standard deviation of a series of readings is a measure of the scatter of the test results about the mean value.

$$\sigma := \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} \quad \text{Formula 6}$$

where:

$\sigma$  is the empirical standard deviation

$\bar{x}$  is the first order expected value (mean value)

$$\bar{x} := \frac{1}{N} \sum_{i=1}^N x_i \quad \text{Formula 7}$$

N is the number of readings

$x_i$  is reading i

### Standard uncertainty

Characterizes the standard deviation of the measurement uncertainty of a characteristic value, which we call the standard uncertainty.

### Combined standard uncertainty

Standard uncertainty of a test result when several mutually independent parameters affect the result. It is equal to the positive square root of the sum of the squares of the individual parameters.

$$u_c := \sqrt{u_1^2 + u_2^2 + \dots + u_n^2} \quad \text{Formula 8}$$

$u_c$  : combined standard uncertainty

### Expanded uncertainty

The standard uncertainty multiplied by a factor k. If k = 2, it can be assumed that the measurement result will lie within this uncertainty range in 95% of all cases.

### Normal distribution

Many natural, scientific and engineering processes can be described either exactly or to a close approximation by a normal distribution function. This also applies when the measurement uncertainty has several causes.

The normal distribution is specified by the probability density function

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \quad \text{Formula 9}$$

where  $\sigma$  is the standard deviation and  $\mu$  the expected value of the normal distribution.

The normal distribution is also called the Gaussian distribution, as its probability density is represented by a Gaussian (bell-shaped) curve that is symmetrical about the expected value  $\mu$  and whose height and width are a function of  $\sigma$ .

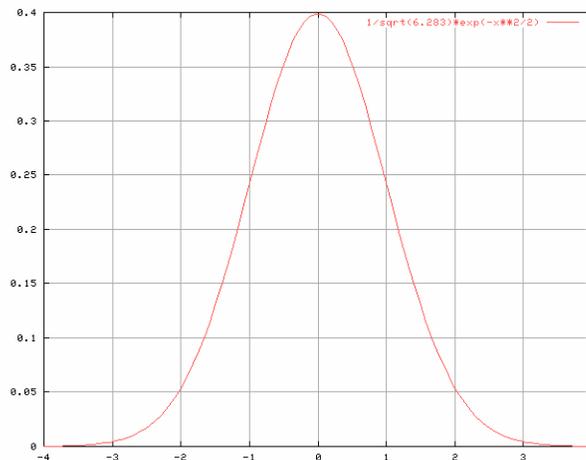


Figure 14: Probability density of the normal distribution

The area under the curve is equal to 1. Incremental areas under the curve correspond to probabilities. The confidence intervals 1, 2 and 3 are marked on the graph. If the uncertainty is specified to be the interval  $[-1, 1]$ , there is a 68% probability that measurement results will lie within the stated interval. The probability increases to 95.5% with the interval  $[-2, 2]$ , and with the interval  $[-3, 3]$  there is a 99.7% probability that the measurement results will lie within the stated interval.

## Uniform distribution

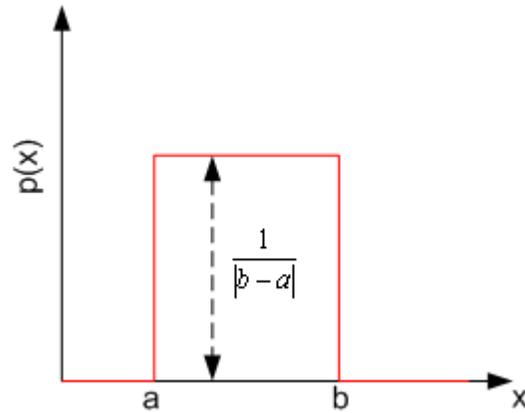


Figure 15: A distribution density function  $p(x)$  with a uniform distribution between limits  $a$  and  $b$

The expected value  $\mu$  of the uniform distribution is  $\mu = \frac{b+a}{2}$ ,

and the standard deviation  $\sigma$  is  $\sigma := \sqrt{\frac{(b-a)^2}{12}}$ . Formula 10

For symmetrical error limits ( $a = -b$ ), the expected value is 0 and the standard deviation  $\sigma$  is as follows

$\sigma := \frac{b}{\sqrt{3}}$  Formula 11

### U distribution

The U-shaped probability distribution occurs with errors due to mismatches. The probability of a measured value close to the maximum value is higher than the probability of a small measured value.

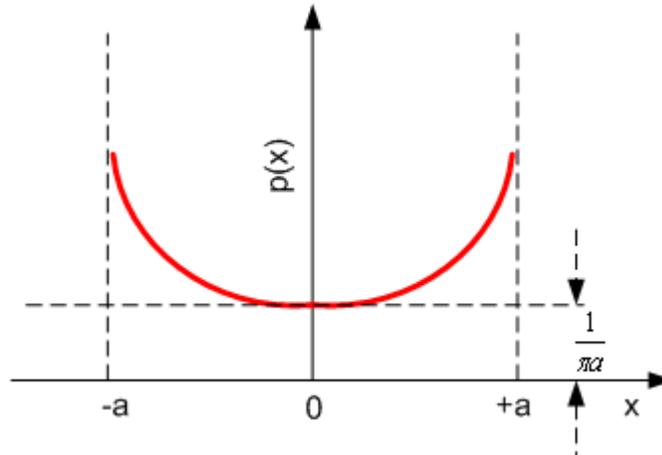


Figure 16: Probability density function  $p(x)$  with a U-shaped distribution

The probability density function is described by

$$p(x) := \frac{1}{\pi\sqrt{a^2 - x^2}} \text{ where } -a < x < +a.$$

The standard deviation is calculated as follows:  $\sigma := \frac{a}{\sqrt{2}}$ .

### Type A measurement uncertainty

The measurement uncertainty due to statistical fluctuations such as thermal noise is called the Type A measurement uncertainty.

### 7 References

1. *Leitfaden zur Angabe der Unsicherheit beim Messen*, DIN (Deutsches Institut für Normung) [German translation of *Guide to the Expression of Uncertainty in Measurement (GUM)*]
2. Measuring Receiver R&S® FSMR – Product Brochure, PD 0758.2319.12, Version 01.00, November 2004
3. Measuring Receiver R&S® FSMR – Specifications, PD 0758.2319.22, Version 2.0, March 2005
4. Power Meter R&S® NRP – Data Sheet, PD 0757.7023.21
5. Power Sensors R&S® NRP-Z51 & NRP-Z55 – Technical Information, TI\_NRP-Z51\_55.doc, September 2003
6. Power Sensor Modules R&S® NRP-Z27 & NRP-Z37 – Technical Information, TI\_NRP-Z27\_37.doc, December 2005
7. ETSI TR 100 028-2 V1.4.1 (2001-12), Annex D

Rohde & Schwarz documentation is available on the Internet at [www.rohde-schwarz.com](http://www.rohde-schwarz.com).

### 8 Additional information

This application note and associated spreadsheets are updated from time to time. Please visit the website **1MA92** to download new versions.

Please contact [TM-Applications@rsd.rohde-schwarz.com](mailto:TM-Applications@rsd.rohde-schwarz.com) if you have any comments or further suggestions.

## 9 Ordering information

<b>Instrument</b>		
Designation	Range	Order No.
R&S FSMR3	20 Hz to 3.6 GHz	1166.3311.03
R&S FSMR26	20 Hz to 26.5 GHz	1166.3311.26
R&S FSMR50	20 Hz to 50 GHz	1166.3311.50
R&S NRP-Z51	1 $\mu$ W to 100 mW, DC to 18 GHz	1138.0005.02
R&S NRP-Z55	1 $\mu$ W to 100 mW, DC to 40 GHz	1138.2008.02
R&S NRP-Z27	4 $\mu$ W to 400 mW, DC to 18 GHz	1169.4102.02
R&S NRP-Z37	4 $\mu$ W to 400 mW, DC to 26.5 GHz	1169.3206.02
<b>Options</b>		
R&S FSMR B23	20 dB preamplifier, 3.6 GHz to 26.5 GHz, for R&S FSMR26	1157.0907.05
R&S FSMR B223	YIG preselector, 3.6 GHz to 26.5 GHz, with 20 dB preamplifier, 3.6 GHz to 26.5 GHz, for R&S FSMR26	1157.1955.26
R&S FSMR B25	Electronic attenuator and 20 dB preamplifier (3.6 GHz)	1044.9298.02



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