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# Producing Epicycloid Traces

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## Application Note 1EZ28\_0L

Subject to change

31 May 1996, Olaf Ostwald

Products:

**ZVR**

**ZVRE**

**ZVRL**



**ROHDE & SCHWARZ**

**This** is a very special application note for all those who have seen the ZVatch (ZVR watch) and not really believed the nice red curve on its face.

This curve can actually be generated with a Vector Network Analyzer ZVR. **As a proof**, the original measurement curve is shown in Fig. 1, which was obtained with a ZVR on December 1, 1995.

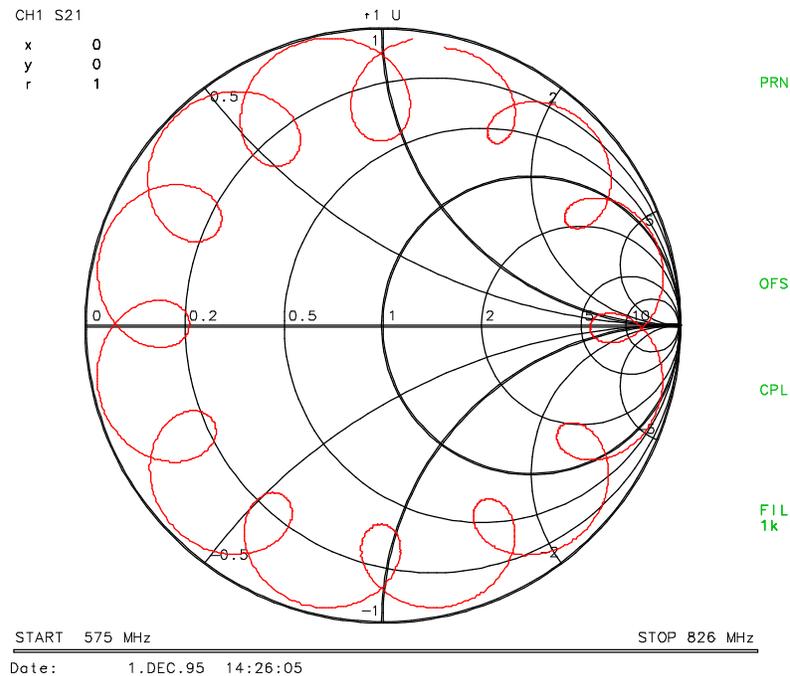


Fig. 1: Original measurement curve used for the ZVatch

## 1 Measurement setup

**To perform this measurement** on your own, you could simply use two power splitters (e.g. RVZ), a delay line (e.g. two or three cables ZV-Z11), a 10-dB-attenuator (e.g. DNF) and some through-connections (e.g. from Cal Kit ZV-Z21). Before starting the measurement a calibration of the Network Analyzer is not necessary but recommended: leave Port 1 open and connect a cable ZV-Z11 to Port 2. After a PRESET of the Analyzer press CAL: START NEW CAL: FULL TWO PORT: N 50Ω FEMALE: N 50Ω MALE: TOM.

Now perform a TOM calibration as usual, connecting the cable directly to Port 1 and connecting the open and match standards of Cal Kit ZV-Z21 to Port 1 and the end of the cable resp., and finally APPLY CAL.

Allow the ZVR to interpolate the calibration data by pressing CAL: CAL INTERPOL. After that, the measurement can be carried out using a test setup as sketched in Fig.2.

Connect the input port of one power splitter to Port 1 of the analyzer. Then join one output port of the power splitter to an output port of the other power splitter using an adapter. The second splitter is used here as a power combiner. Connect the other output port of the first splitter with the attenuator and the long cable to the output port of the second power splitter. Finally connect the input port of the second power splitter to the cable at Port 2.

Now a measurement of the transmission parameter S21, for instance with CENTER: 1 GHz and SPAN: 1 GHz, should yield a curve on the display similar to Fig.1. Fine tuning of the curve can be done by varying CENTER frequency and SPAN as well as optimizing MAGNITUDE and PHASE OFFSET and ELECTRICAL LENGTH OFFSET. For changing the colour of the curve press DISPLAY: CONFIG DISPLAY: COLORS and turn TINT, BRIGHTNESS and SATURATION e.g. all to 100%, which yields a bright red trace.

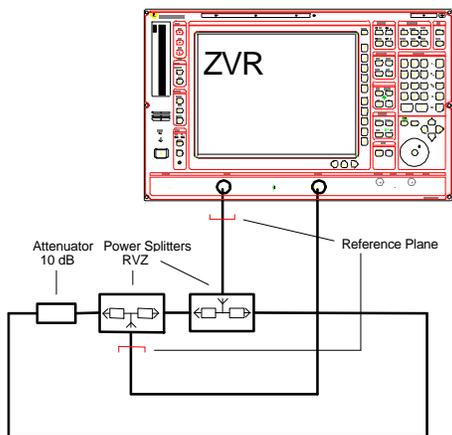


Fig. 2: Test setup for the measurement

Instead of the described transmission measurement, alternatively a reflection measurement can be performed. The first has the practical advantage of a higher accuracy, while the latter may be simpler to carry out. For that, one needs only one power splitter, which is connected to Port 1 of the analyzer. One of the output ports of the power splitter may be left open, while a cable and a 6-dB-attenuator should be connected to the other output port of the power splitter.

## 2 Theory

**The theoretical background** of this special type of curve is a superposition of the transfer functions  $H_1(\omega)$  and  $H_2(\omega)$  of two delay lines, having different attenuations  $A_1$  and  $A_2$  and delays  $\tau_1$  and  $\tau_2$ . The superposition can be described as the sum

$$H(\omega) = A_1 \cdot e^{-j\omega\tau_1} + A_2 \cdot e^{-j\omega\tau_2}$$

This equation can be interpreted as the vector sum of two vectors as illustrated in Fig. 3, one with amplitude  $A_1$  and phase  $-\omega\tau_1$ , the other with amplitude  $A_2$  and phase  $-\omega\tau_2$ . The two vectors rotate proportionally to frequency  $\omega$ , but generally with different speeds. Assuming  $\tau_2 > \tau_1$ , which is produced by the long cable versus the short adapter between the two power splitters, and  $A_2 < A_1$  caused by the 10-dB-attenuator, the first vector rotates more slowly with increasing frequency than the other, while the latter has a shorter length than the first vector. Each of them describes a circle in polar coordinates.

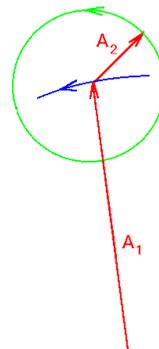


Fig. 3: Superposition of two rotating vectors

As a result, the superposition of the two circles describes a mathematical curve, called an epicycloid, which is produced if one circle rolls on another circle. This ingenious combination of circular motions was devised by the greek astronomer Claudius Ptolemy in order to explain the apparently extremely complex motions of the planets around the earth. Thus the Vector Network Analyzer ZVR takes up a fourteen centuries lasting tradition of aesthetic results based on innovative ideas and fine measuring equipment.

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